Characterizing work in complex, chaotic quantum systems

Aurélia Chenu

June 27th, 2019

AC, I. Egusquiza, J. Molina-Vilaplana, A. del Campo, Sci. Rep. 8:12634 (2018)

AC, J. Molina-Vilaplana, A. del Campo, Quantum 3:127 (2019)

1. Work is not an observable $\langle W \rangle = \int p(W) W dW$

Driven Isolated System

$$H_s = \sum_n E_n^s |n_s\rangle\langle n_s|$$

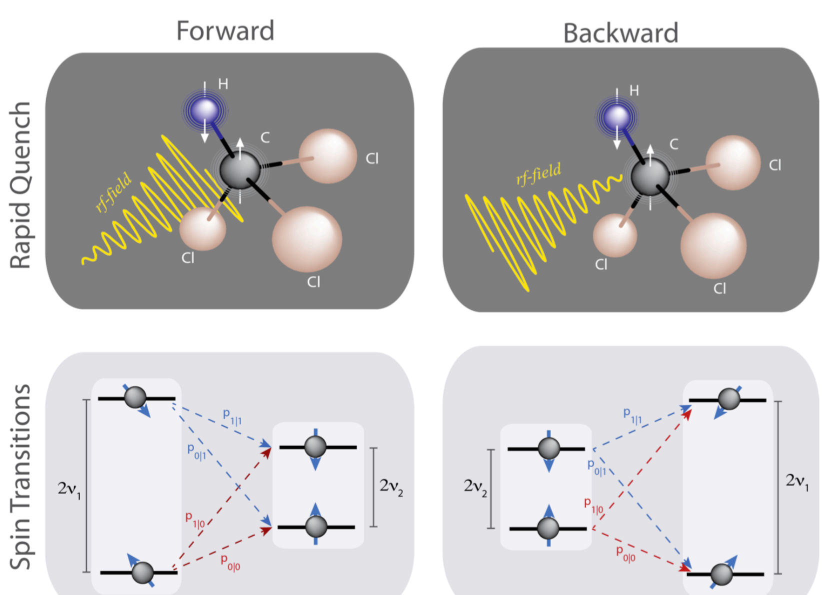
$$U(\tau) = \mathcal{T} \exp\left[-i \int_0^\tau ds \hat{H}_s\right]$$

Work requires two projective measurements

J. Kurchan ArXiv:0007360
P. Talkner, E. Lutz, P. Hänggi PRE (2007)

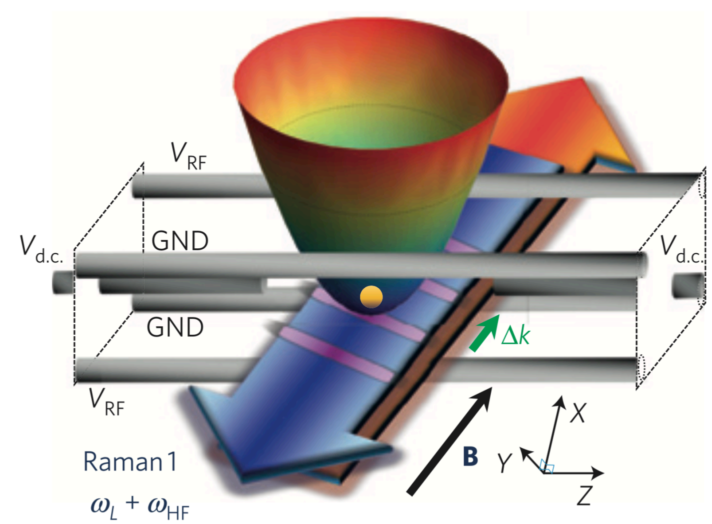
$$p_\tau(W) := \sum_{n,m} p_n^0 \, p_{m|n}^\tau \delta\left[W - \left(E_m^\tau - E_n^0\right)\right]$$

Experimentally measured in simple quantum systems



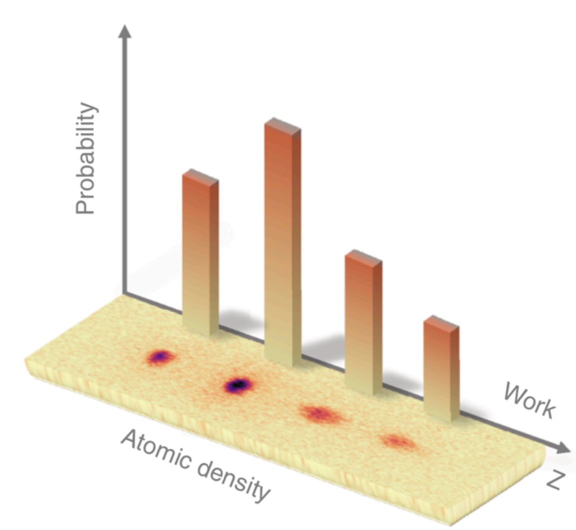
NMR setting

Batalhão et al. PRL (2014)



Driven oscillator (trapped ion)

S. An et al. Nat. Phys. (2015)



Ultracold atom

F. Cerisola et al. Nat. Comm. (2017)

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2. A DYNAMICAL APPROACH TO WORK

$$p_\tau(W) := \sum_{n,m} p_n^0 \, p_{m|n}^\tau \delta \left[W - \left(E_m^\tau - E_n^0 \right) \right] = \frac{1}{2\pi} \int_{\infty} dt \, \chi(t,\tau) e^{-itW}$$

Recast the characteristic function of the work distribution function as a Loschmidt echo amplitude

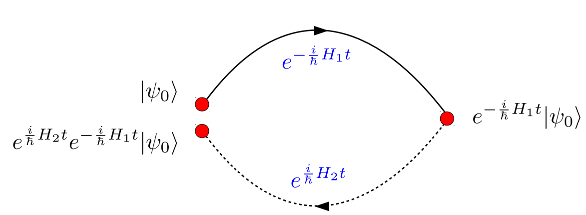
$$\chi(t,\tau) = \mathrm{Tr} \left(\mathrm{U}^\dagger(\tau) \mathrm{e}^{\mathrm{itH}_\tau} \mathrm{U}(\tau) \, \mathrm{e}^{-\mathrm{itH}_0} \rho_{\mathrm{mix}} \right)$$

For pure state $|\Psi_0\rangle = |n_0\rangle$

Silva PRL (2008)

$$\chi(t,0^+) = \langle j_0 | e^{+itH_\tau} e^{-itH_0} | j_0 \rangle$$

$$= \langle \Psi_t | \Psi_0 \rangle$$

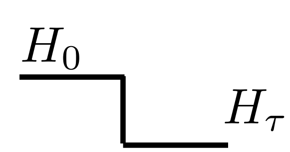


For mixed state $|\Psi_0\rangle = \sum_n \sqrt{p_n^0} \, |n_0\rangle_L \otimes |n_0\rangle_R$

$$\chi(t,\tau) = \langle \Psi_0 | e^{+itH_\tau^{\mathrm{eff}}} e^{-itH_0} \mathbf{1}_R | \Psi_0 \rangle$$

$$\boxed{\chi(t,\tau) = \langle \Psi_t | \Psi_0 \rangle}$$

Sudden Quench



$$H(t) = H_0 \, \Theta(-t) - H_\tau \, \Theta(t)$$

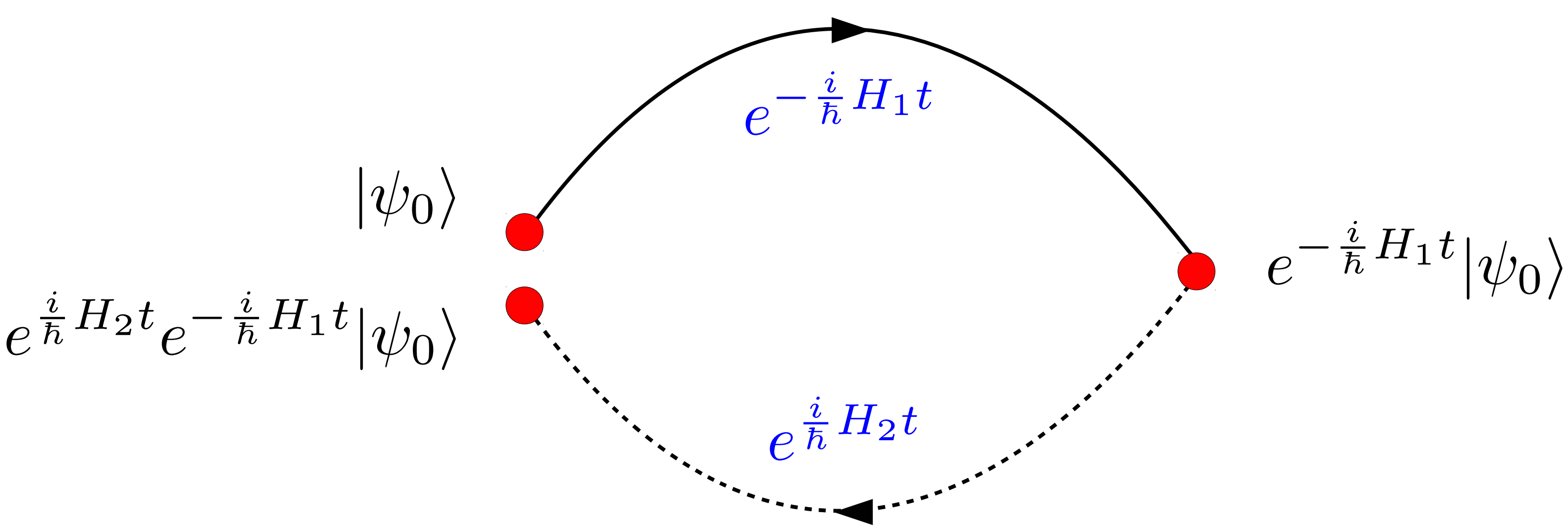
AC, I. Egusquiza, J. Molina-Vilaplana, A. del Campo, Sci. Rep. 8:12634 (2018)

LOSCHMIDT ECHO AND WORK FLUCTUATIONS

$$\boxed{\chi(t,\tau) = \langle \Psi_t | \Psi_0 \rangle}$$

$$\boxed{\mathcal{L}(t) = |\langle \Psi_t | \Psi_0 \rangle|^2}$$

$$= e^{-\sigma_W^2 t^2 + \mathcal{O}(t^4)}$$



A measure of irreversibility

Josef Loschmidt (1821 - 1895)

3. Information Scrambling

Lectures on Gravity and Entanglement,
M. Van Raamsdonket (2015)

Thermofield double field: purified thermal state

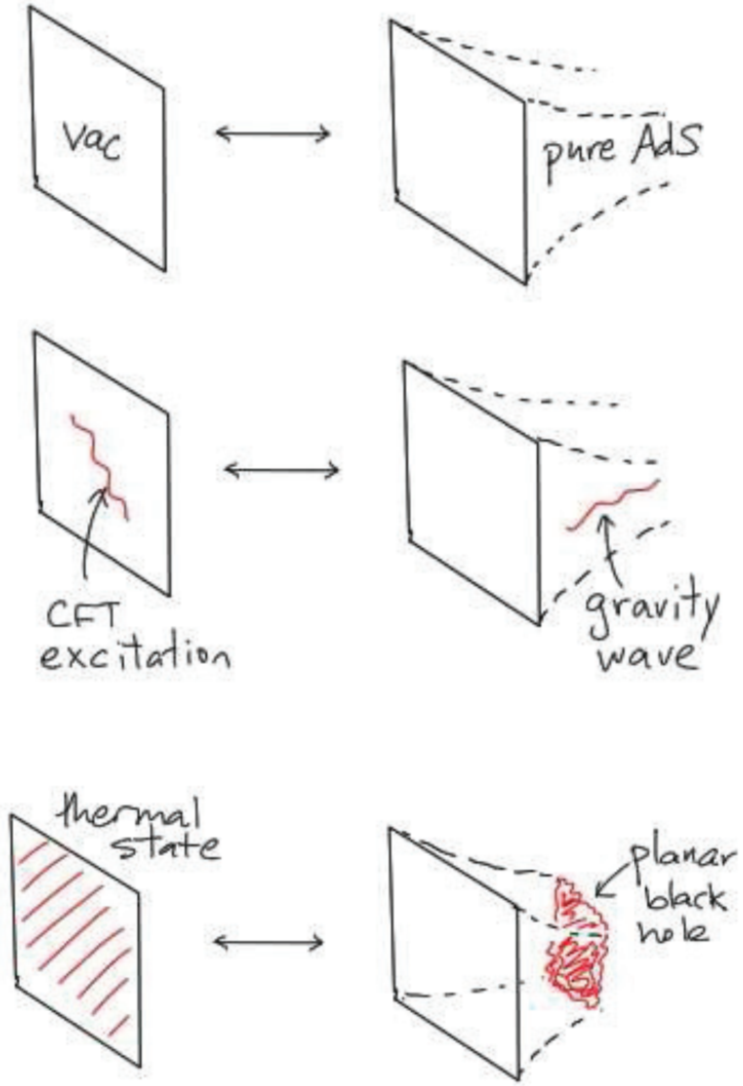
$$|\Psi_0\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_n e^{-\frac{\beta}{2}E_n} \, |n\rangle_L \otimes |n\rangle_R$$

$$\Big\downarrow \; H_L \otimes \mathbf{1}_R$$

del Campo et al.
PRD (2018)

$$\langle \Psi_t | \Psi_0 \rangle = \frac{Z(\beta + it)}{Z(\beta)}$$

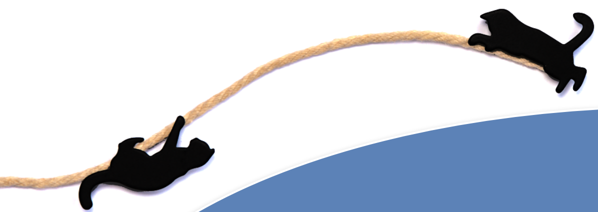
CFT AdS



The TDF is dual to an eternal BH
(Maldacena 2003)

Loschmidt echo from analytical continuation of the partition function

$$\mathcal{L}(t) = |\langle \Psi_t | \Psi_0 \rangle|^2 = \left| \frac{Z(\beta + it)}{Z(\beta)} \right|^2$$



Work Statistics

$$p(W) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dt\, \chi(t,\tau) e^{-itW}$$

Loschmidt Echo

$$\mathcal{L}(t) = |\chi(t,0^+)|^2$$

Information Scrambling

$$\frac{Z\left(\beta + i2t\right)}{Z\left(\beta\right)} = \chi(t,0^+)$$

AC, I. Egusquiza, J. Molina-Vilaplana, A. del Campo, Sci. Rep. 8:12634 (2018)

Work Statistics of Chaotic Quantum Systems

AC, J. Molina-Vilaplana, A. del Campo, Quantum 3:127 (2019)

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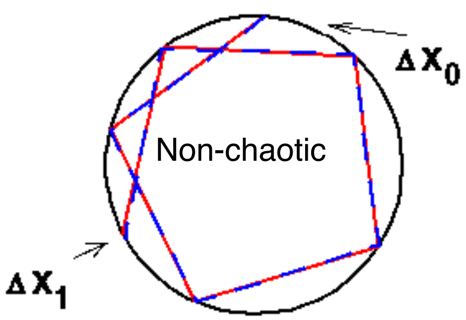
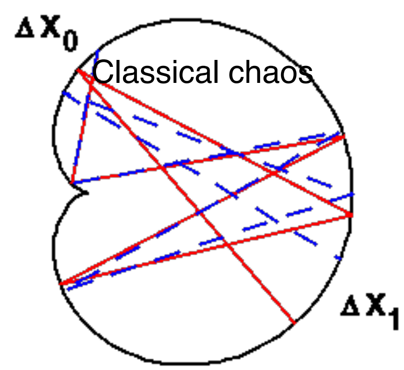
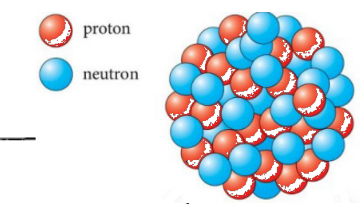
Chaotic systems as paradigmatic complex systems





e.g.



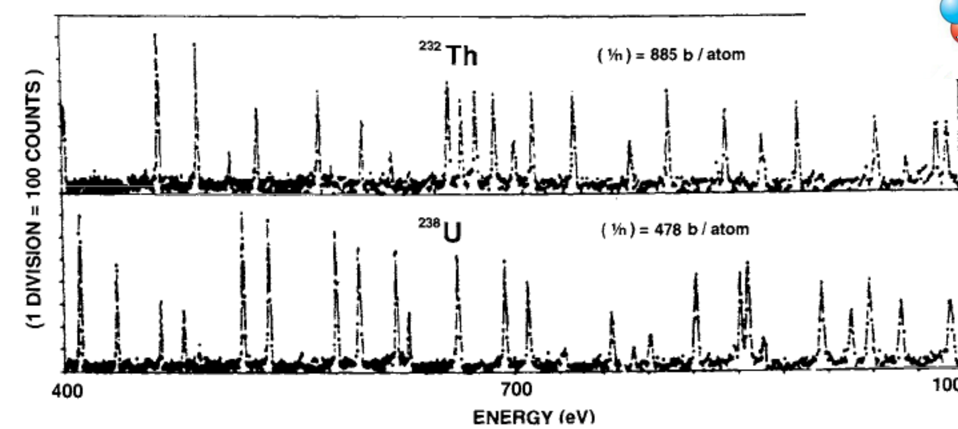
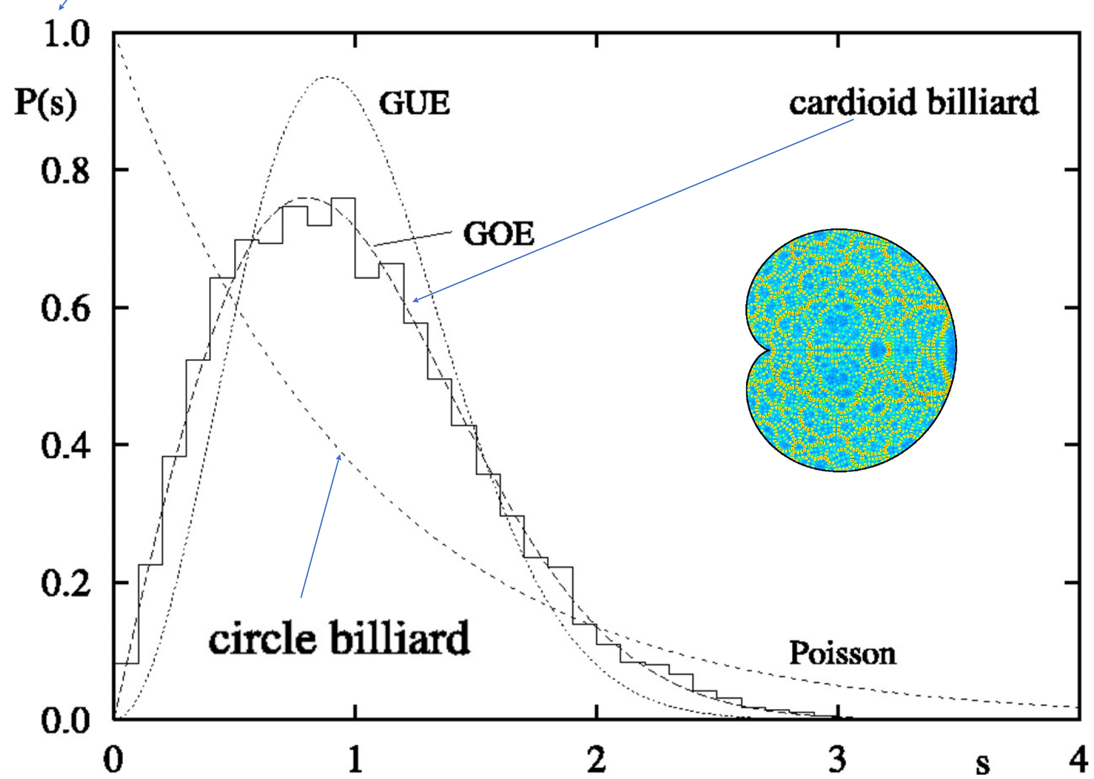
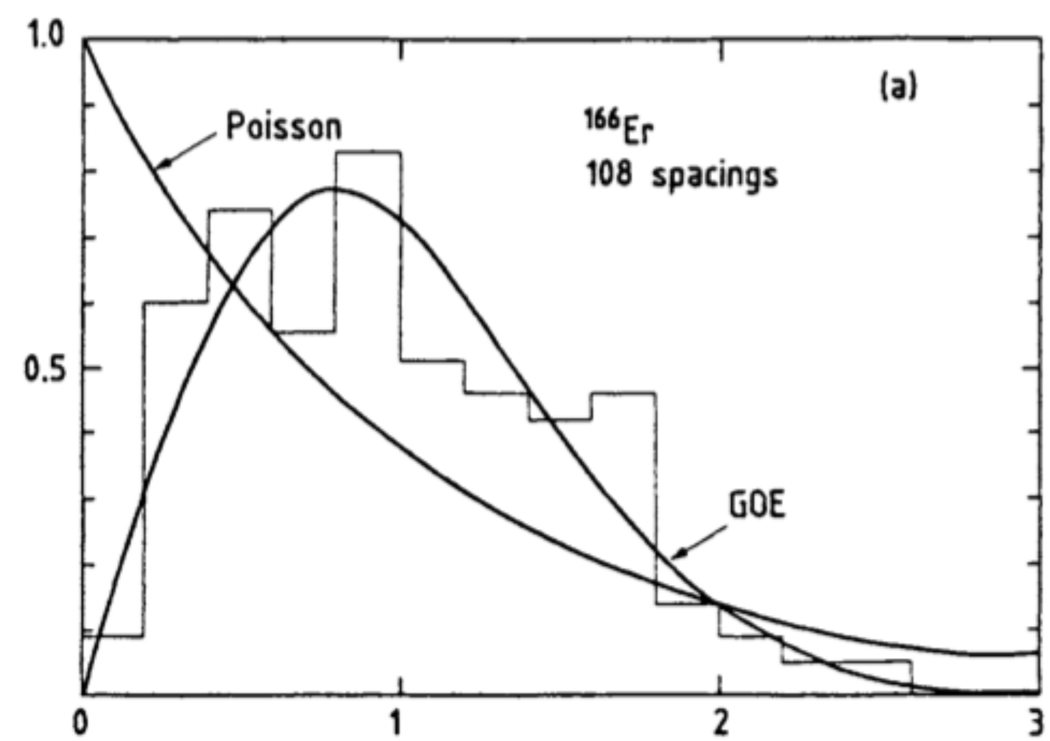


Figure 1.1. Slow neutron resonance cross-sections on thorium 232 and uranium 238 nuclei. Reprinted with permission from The American Physical Society, Rahn et al., Neutron resonance spectroscopy, X, *Phys. Rev. C* 6, 1854–1869 (1972).

P(s) : the pdf of the spacing s between the two consecutive energy levels.





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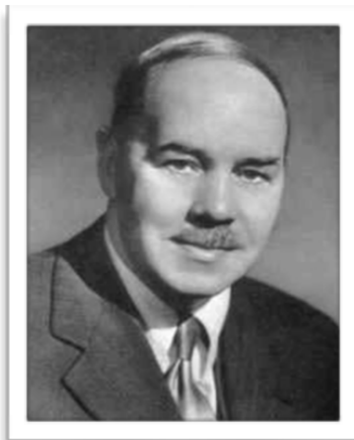
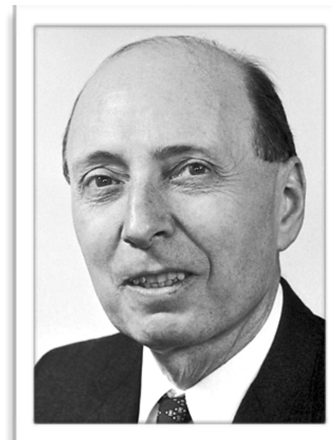
Random matrices and quantum chaos
T. Kriecherbauer, J. Marklof, and A. Soshnikov
PNAS **98** (19) 10531-10532 (2001).

RANDOM MATRICES THEORY (RMT)



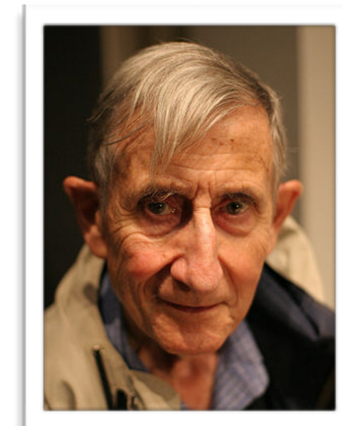
John Wishart

Random Matrices (1928)



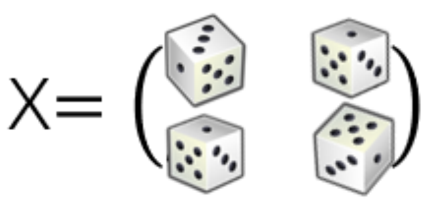
Eugene Paul Wigner

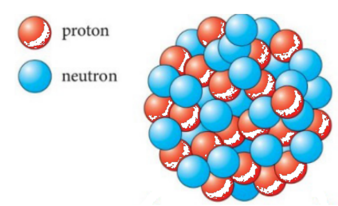
Random Matrices
Ensembles of Hamiltonian (1955)

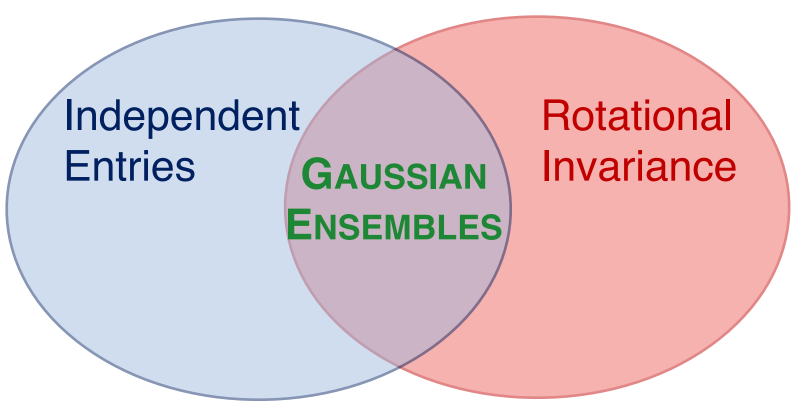


Freeman Dyson

Ensembles of Hamiltonian
GOE, GUE, GSE (1962)







$$\rho[\mathrm{M}] = \rho[\mathrm{UMU}^{-1}]$$

	M	U
GOE	Real symmetric	Orthogonal
GUE	Complex hermitian	Unitary
GSE	Quaternion selfdual	Symplectic

Wigner's Semicircle
Gaussian Unitary Ensembles (GUE)

RANDOM MATRICES THEORY (RMT)

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Journal of Physics A: Mathematical and Theoretical

Random Matrices: the first 90 years

Guest Editors

Giulio Biroli CEA - Paris, France
Zdzisław Burda AGH, Kraków, Poland
Pierpaolo Vivo King's College London, UK

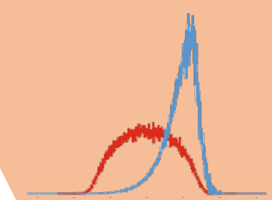
Scope

2018 marks the 90th birthday of Random Matrix Theory (RMT). Originally introduced in multivariate statistics by Wishart, random matrices were first employed in physics at the end of the 50s to model Hamiltonians of heavy nuclei. In recent years, RMT techniques and insights have proven invaluable in deepening our understanding of:

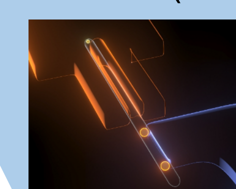
- Fermions in harmonic traps
- Models of interface growth (e.g. KPZ equation and Tracy-Widom), also from the experimental point of view
- Models of delocalized, non-ergodic phases in quantum systems
- Financial applications, like cleaning of correlation matrices
- Condensed matter, like topological insulators
- Stability of ecosystems
- Wireless telecommunications

Complex open quantum systems

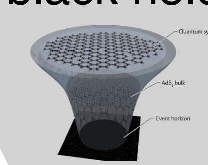
Work statistics

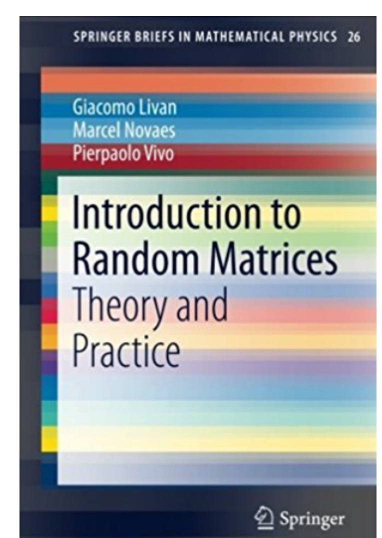


Majorana fermions (SYK)



Information scrambling in black holes

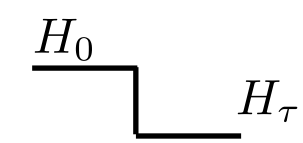




Work statistics in Chaotic Quantum Systems 1/2

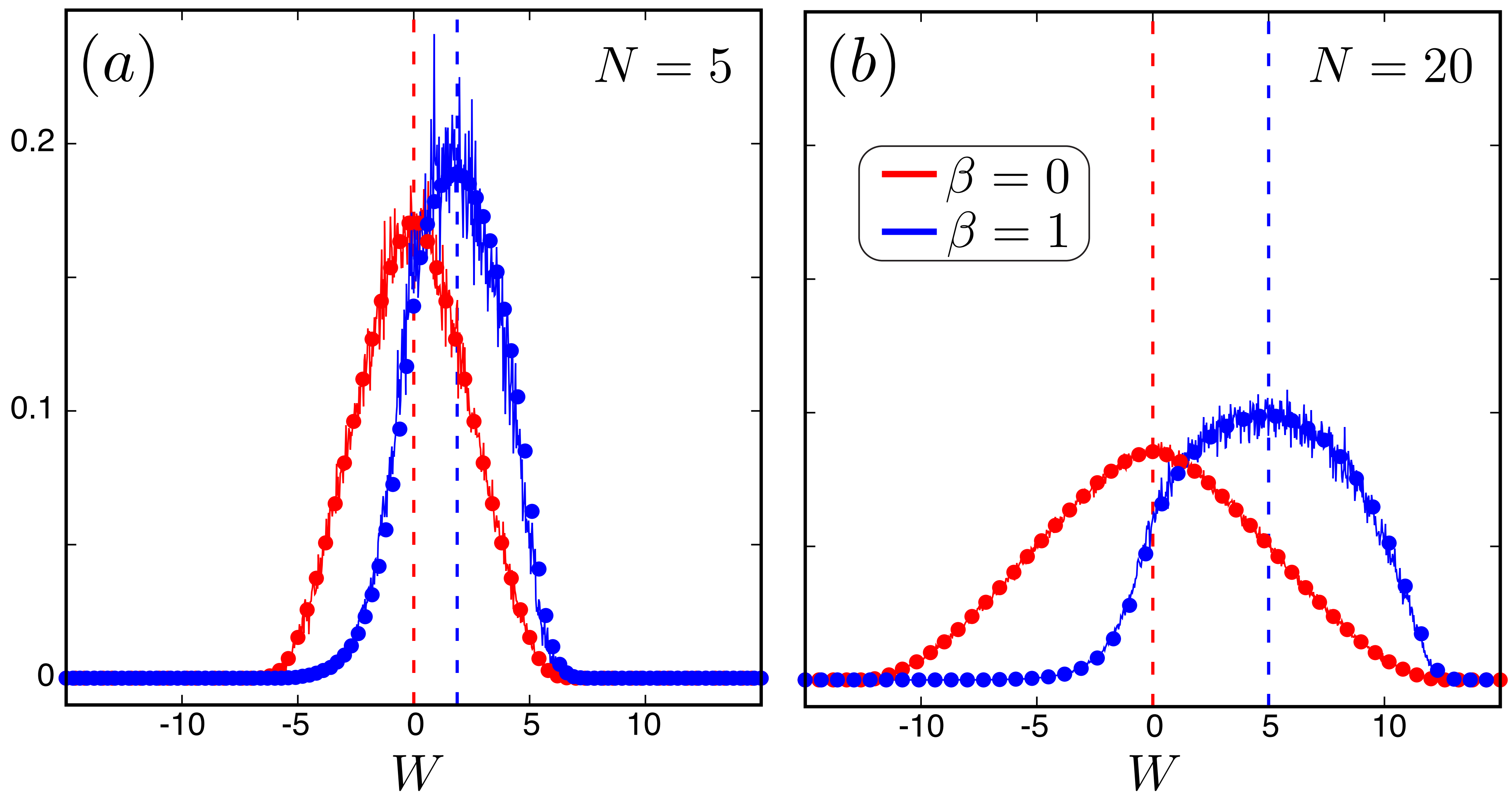
Characteristic function $\langle \chi(t,\tau)\rangle = \frac{1}{\langle Z(\beta)\rangle}\Big\langle\Big\langle \sum_{n,m} |\langle m_\tau|n_0\rangle|^2 e^{-\sigma_\tau E_m^\tau} e^{-\sigma_0 E_n^0}\Big\rangle\Big\rangle$

Sudden Quench



Work PDF

$$\langle p(W)\rangle = \frac{1}{N\mathcal{I}(\beta)}\int_{-\infty}^{\infty} dE\langle\rho(E)\rangle\langle\rho(E+W)\rangle e^{-\beta E}$$



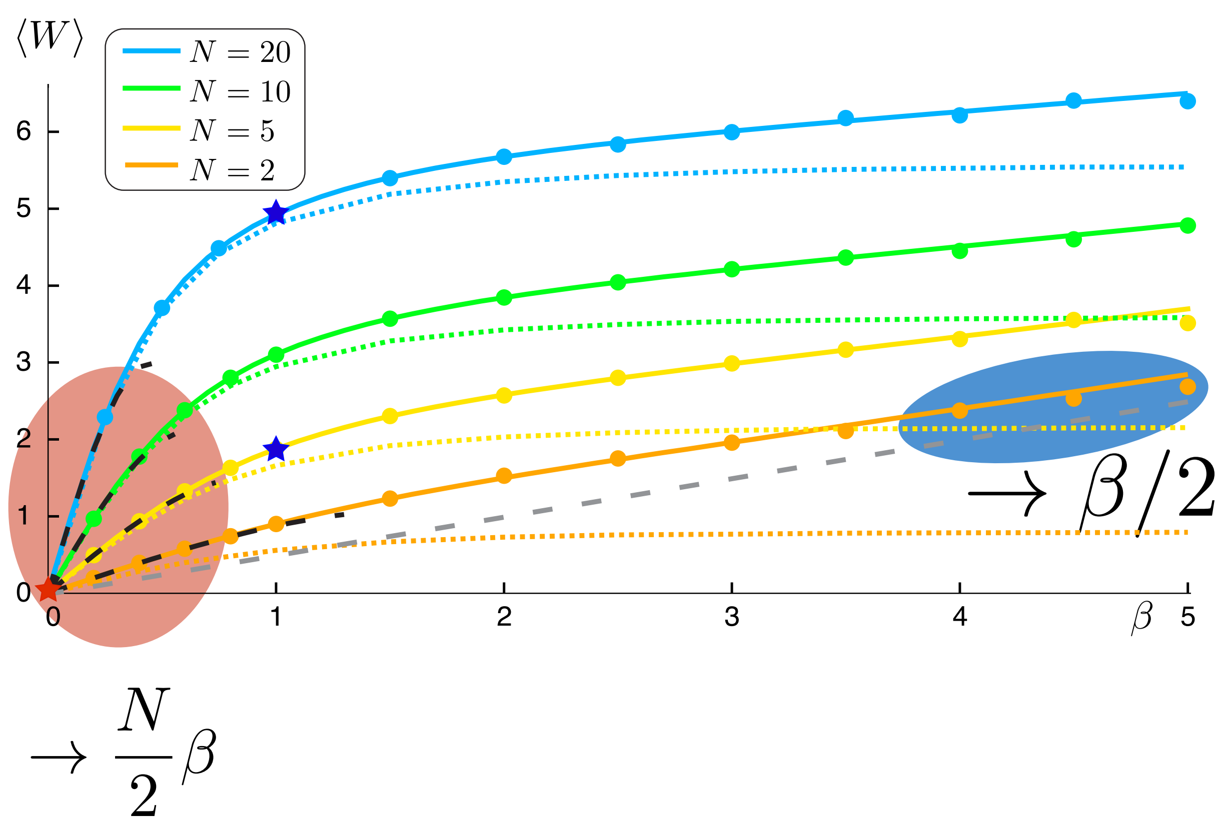
AC, J. Molina-Vilaplana, A. del Campo, Quantum 3:127 (2019)

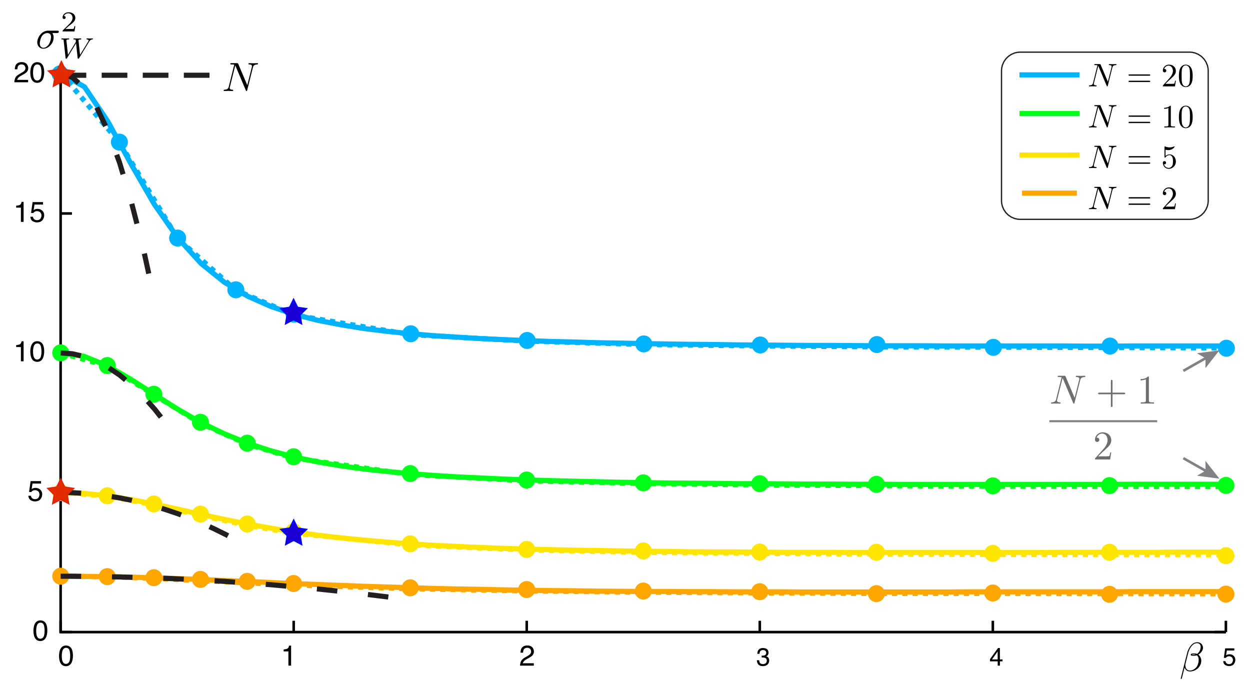
See related work:
RMT large N asymptotics: M. Łobejko, J. Łuczka, P. Talkner PRE 95, 052137 (2017)
Disordered many-body systems: Y Zheng and D. Poletti, arXiv:1806.02555

Work statistics 2/2: mean and fluctuations

$$\langle W \rangle = \langle \hat{H}_\tau \rangle - \langle \hat{H}_0 \rangle$$

$$\sigma_W^2 = \langle W^2 \rangle - \langle W \rangle^2$$





Fluctuations monotonically decay from the infinite-temperature value N, saturating at (N + 1)/2 in the low-temperature regime.

AC, J. Molina-Vilaplana, A. del Campo, Quantum 3:127 (2019)

Loschmidt Echo in chaotic quantum systems

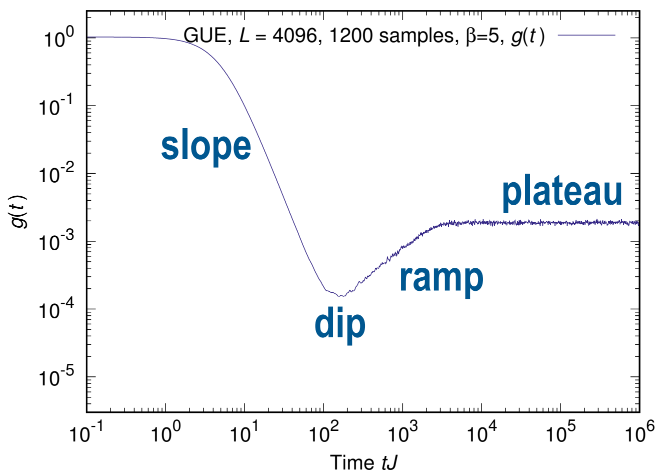
$$\mathcal{L}(t) = |\langle \Psi_t | \Psi_0 \rangle|^2 = \frac{1}{\langle Z(\beta)^2 \rangle} \left\langle \left\langle \left| \mathrm{Tr}\left(e^{-\sigma_\tau \hat{H}_\tau} e^{-\sigma_0 \hat{H}_0} \right) \right|^2 \right\rangle \right\rangle$$

$$= \frac{1}{\langle Z(\beta)^2 \rangle} \frac{1}{N^2-1} \Big(g(0,t) g(\beta,t) + N \langle Z(2\beta) \rangle - \frac{1}{N} \langle Z(2\beta) \rangle g(0,t) - \frac{1}{N} g(\beta,t) N \Big)$$

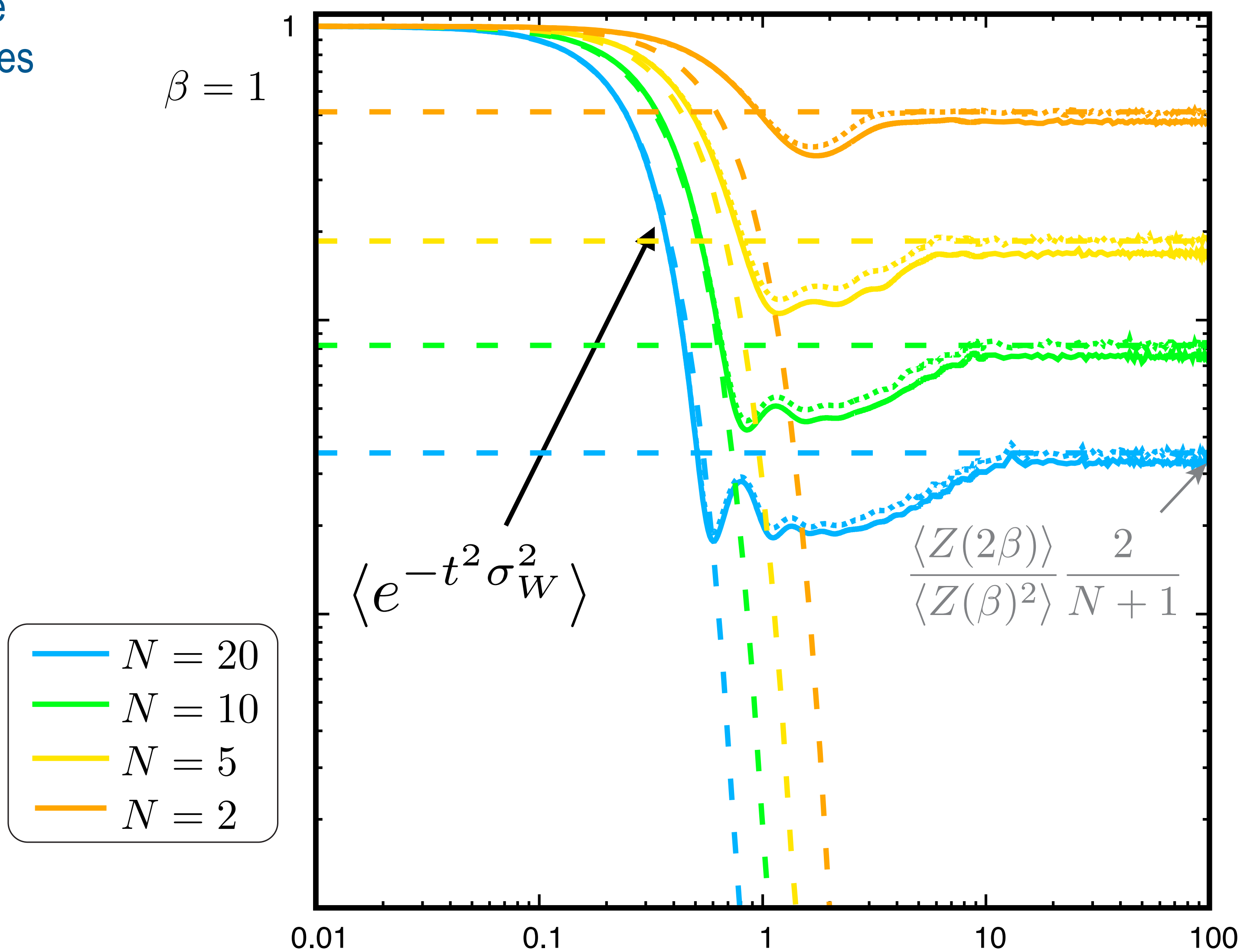
Dependence on the spectral form factor illustrates the importance of correlations between energy eigenvalues

GUE spectral form factor

$$g(\beta,t) \equiv \langle |Z(\beta+it)|^2 \rangle$$



Black holes and random matrices
J. S. Cotler *et al.*, JHEP (2017)



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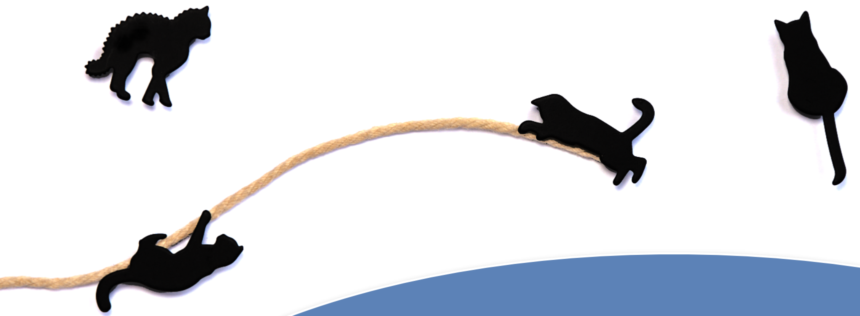
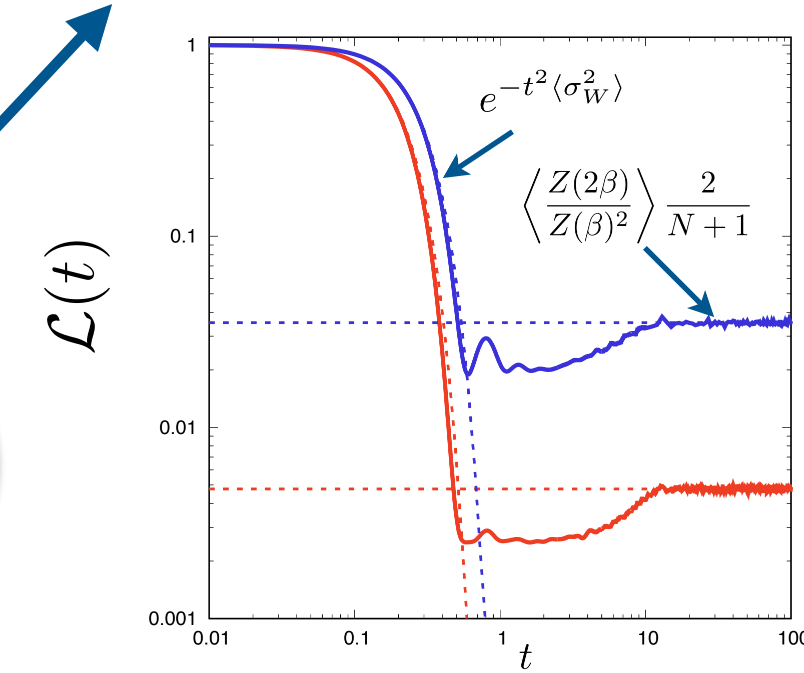


Work Statistics

$$p(W) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dt\,\chi(t,\tau)e^{-itW}$$

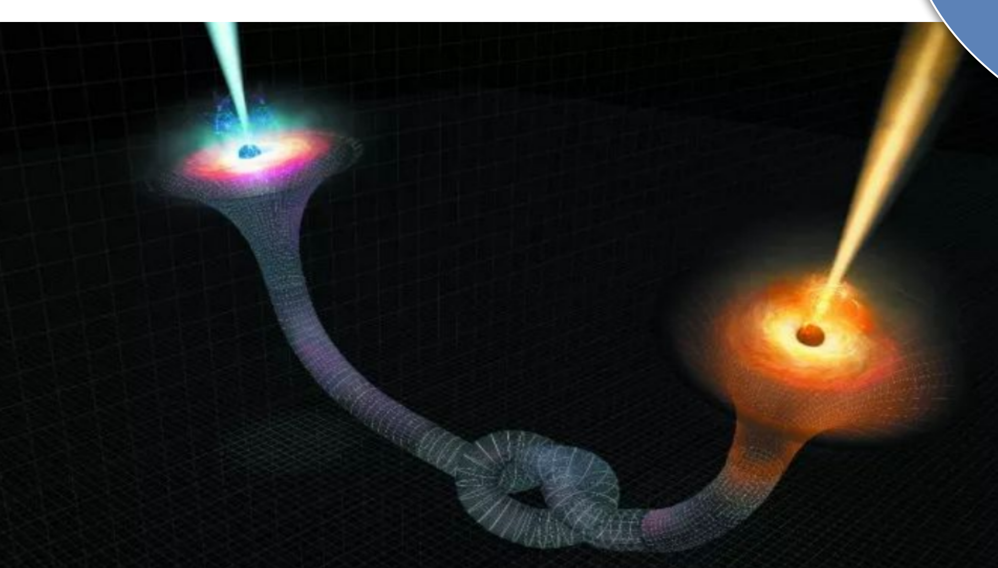
Loschmidt Echo

$$\mathcal{L}(t) = |\chi(t,0^+)|^2$$



Information Scrambling

$$\frac{Z\left(\beta+i2t\right)}{Z\left(\beta\right)} = \chi(t,0^+)$$



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