Ideal projective measurements have infinite resource costs

Yelena Guryanova, Nicolai Friis and Marcus Huber

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IQOQI Vienna, Austrian Academy of Sciences

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Quantum measurements

- Described by POVMs

$$\{M_i\}_i \quad \sum_i M_i^\dagger M_i = \mathbb{I}$$

- State after the measurement

$$\rho_i' = \frac{M_i \rho M_i \dagger}{tr[M_i \rho M_i^\dagger]}$$

- Probability

$$tr[M_i \rho M_i^\dagger]$$

- Projective measurement

$$M_i^\dagger M_i = \Pi_i = \Pi_i^2$$

Thermodynamics from a quantum perspective

- Global reversible time evolution

$$\rho(t) = U\rho_0 U^\dagger$$

- Emergence of thermal states through entanglement and course graining [1,2]

$$\tau(\beta) := \frac{e^{-\beta H}}{\mathcal{Z}(\beta)}$$

- Unitary orbit of thermal states is full rank → third law [3]

[1] M. Gring, et. al, Relaxation and Pre-thermalization in an Isolated Quantum System, Science, Vol. 337 no. 6100 pp. 1318-1322 (2012)
[2] J. Eisert, M. Friesdorf & C. Gogolin, Quantum many-body systems out of equilibrium, Nature Physics volume 11, pages 124–130 (2015)
[3] H. Wilming and R. Gallego, Third Law of Thermodynamics as a Single Inequality, Phys. Rev. X 7, 041033 (2017)

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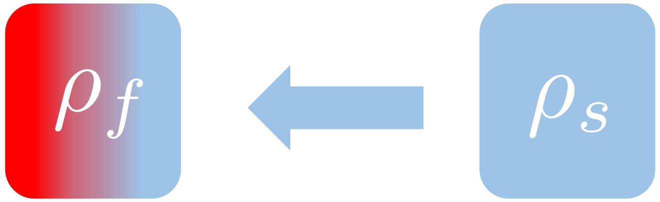
Unattainability principle: it is impossible to cool down to absolute zero with finite resources.

Contradiction?



Cooling a thermal state

- Fridge coupled to target system directly (laser cooling), or bulk (dilution, etc.)
- Work invested, target system gets closer to ground state
- Limited by third law

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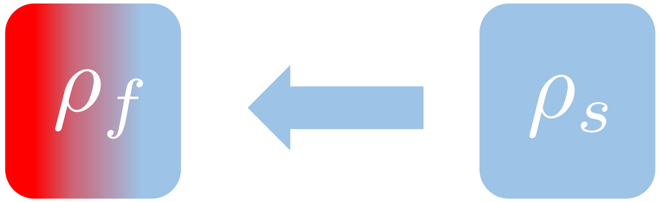

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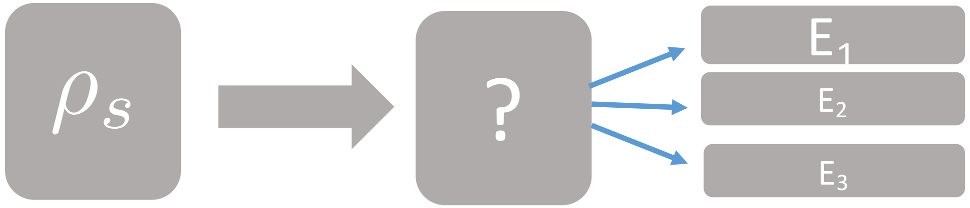

Measuring a thermal state

- Measurement apparatus coupled to target system.
- Measurement in energy eigenbasis. Outcome $|E_i\rangle\langle E_i|$ with probability $\frac{e^{-\beta E_i}}{\mathcal{Z}(\beta)}$
- Limitations?

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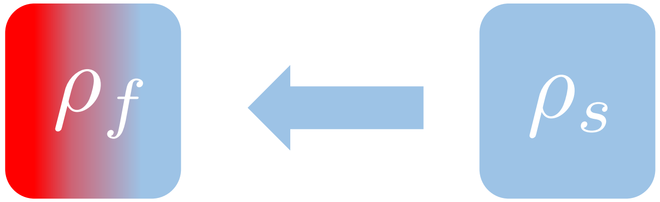

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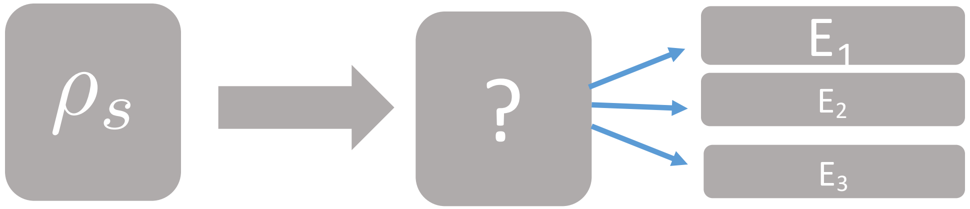

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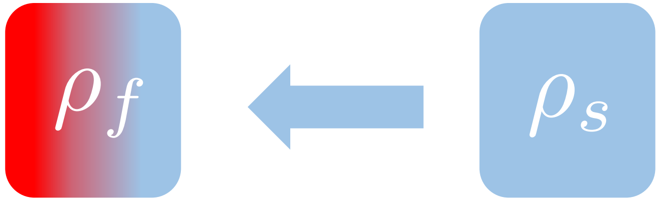

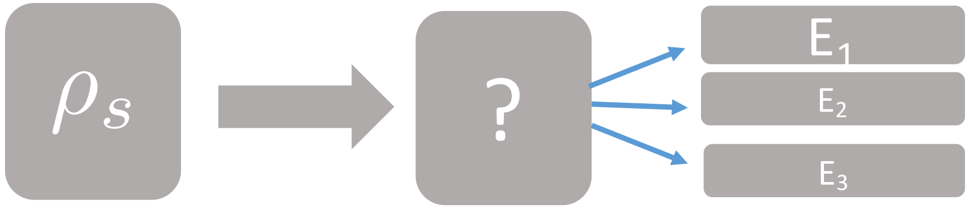

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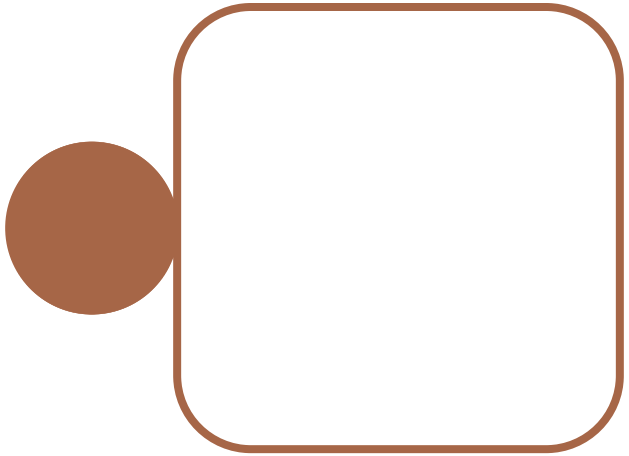
How can a measurement leave the system in a state forbidden by the third law?

Outline

- Quantum measurement
  - Description/ definition
- Ideal measurement
  - 3 mathematical properties
  - Qubit example
- Results
  - Ideal measurements are not possible
  - Approximations to ideal measurements
  - Quantifying the energetic cost of a measurement
- Summary and future works

Measurement

$$\rho_S \otimes \rho_P \longrightarrow \tilde{\rho}_{SP}$$



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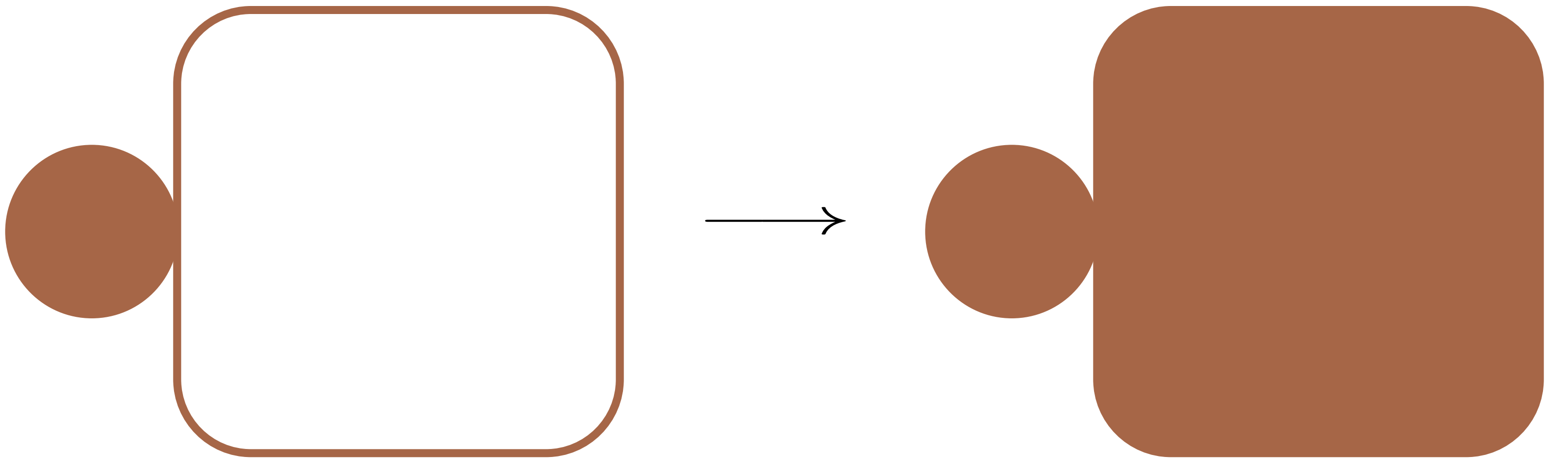

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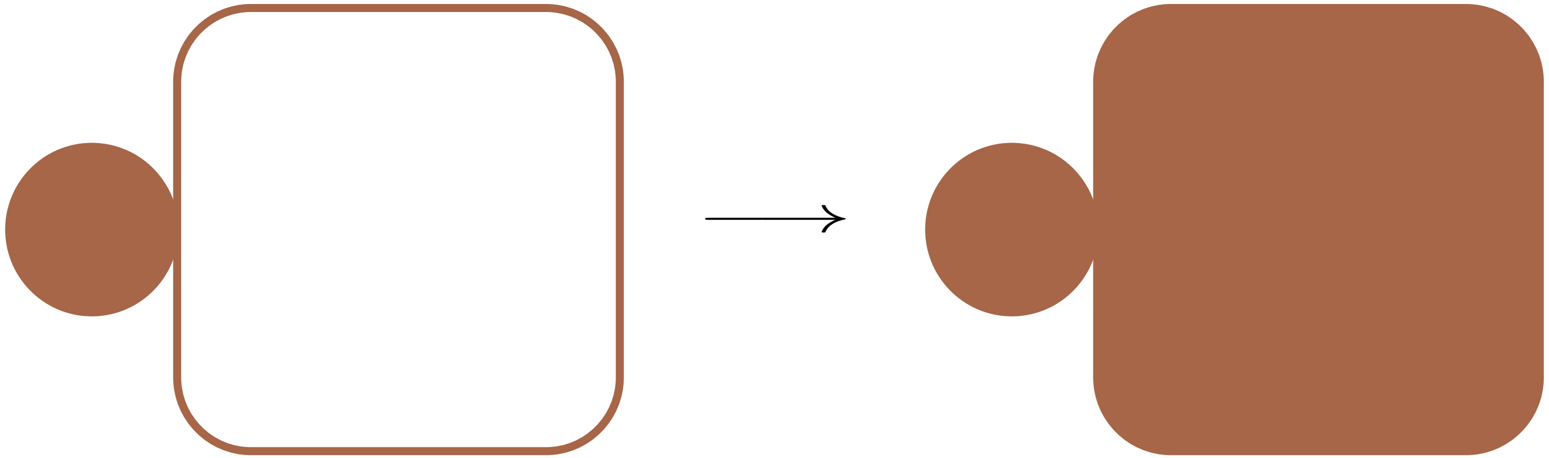

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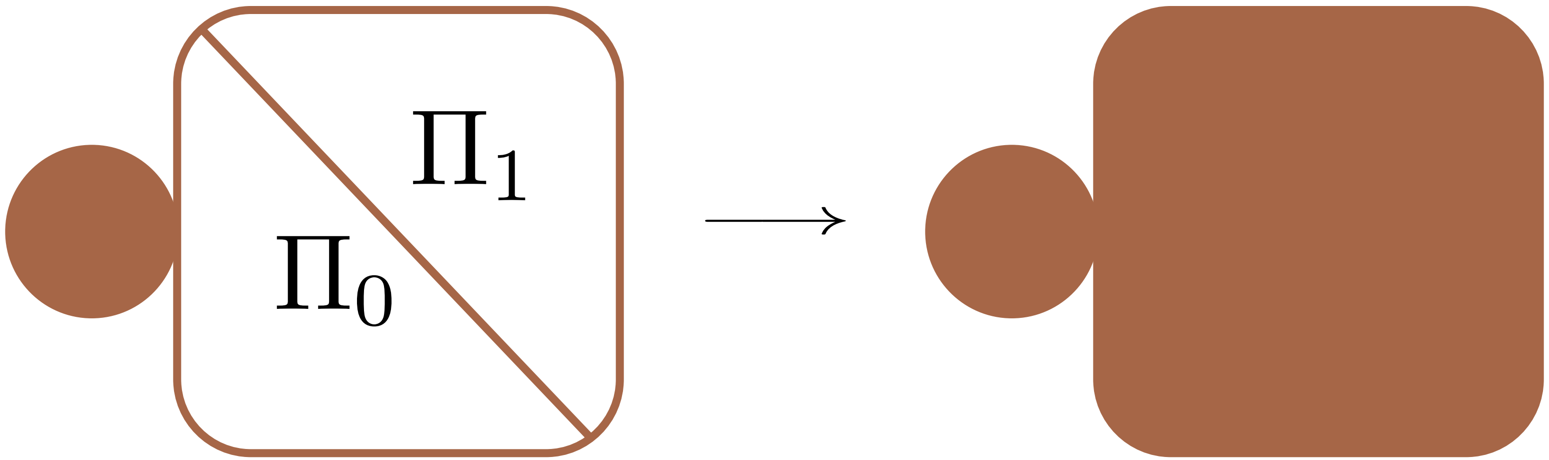

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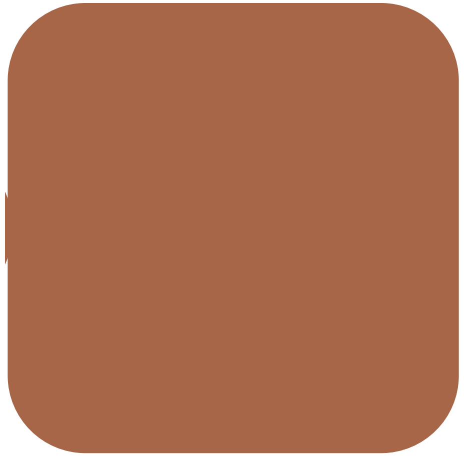

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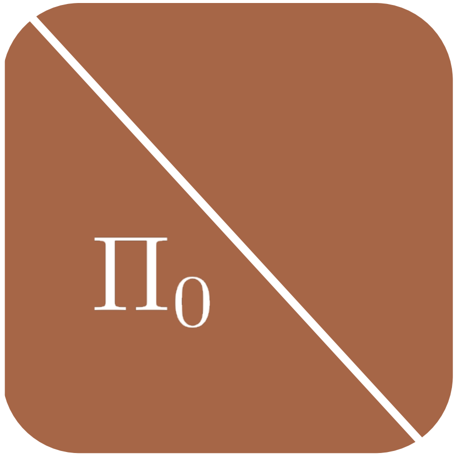

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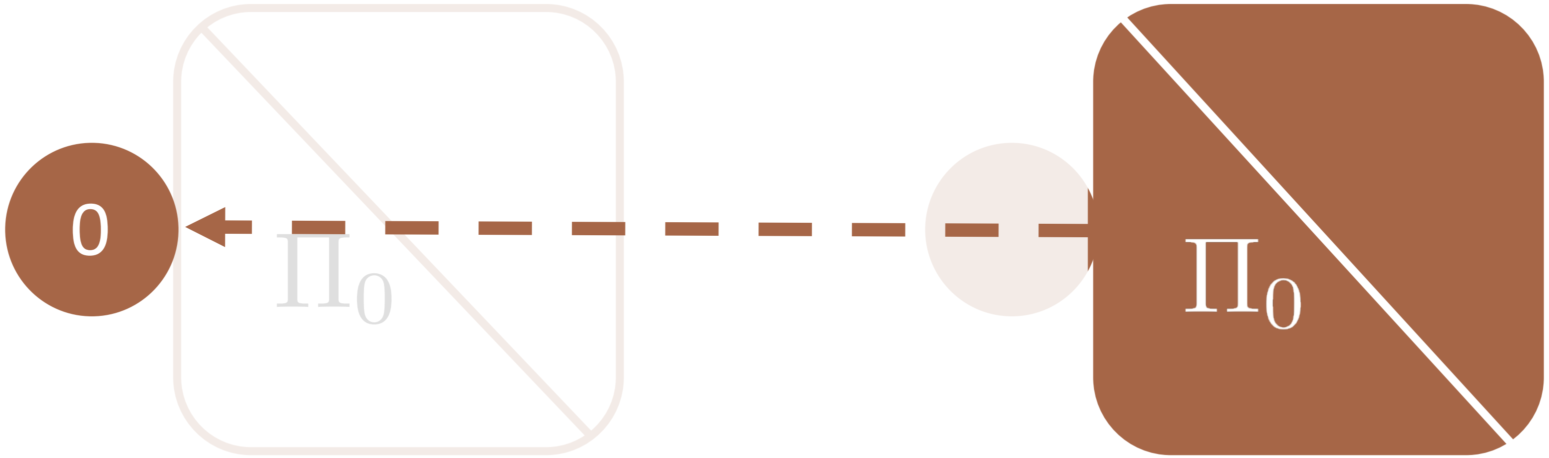

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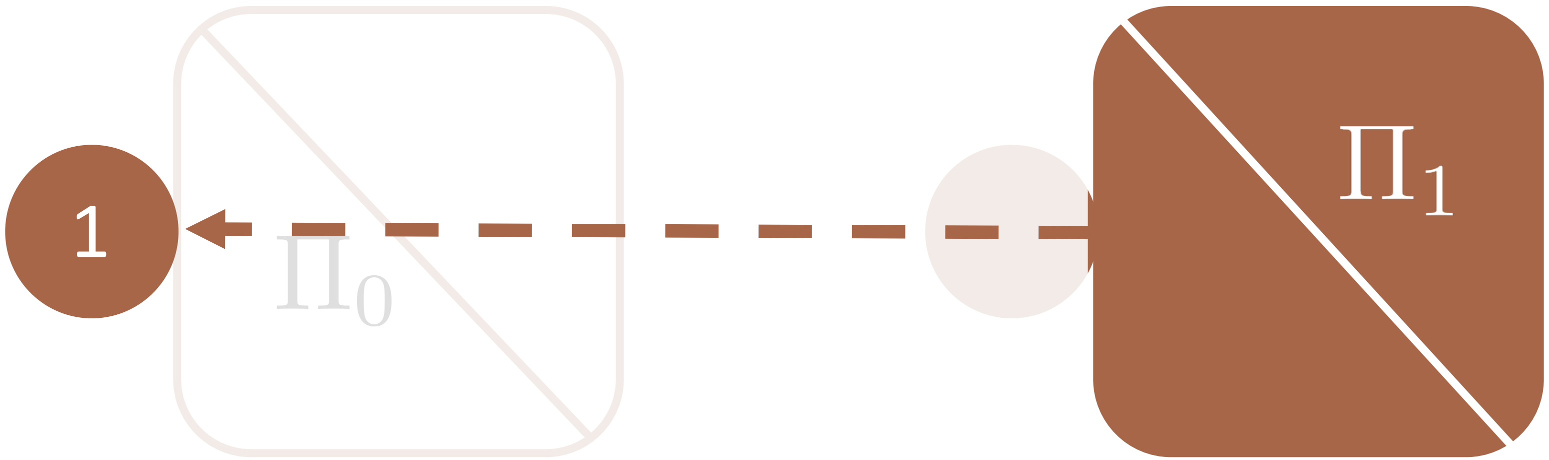

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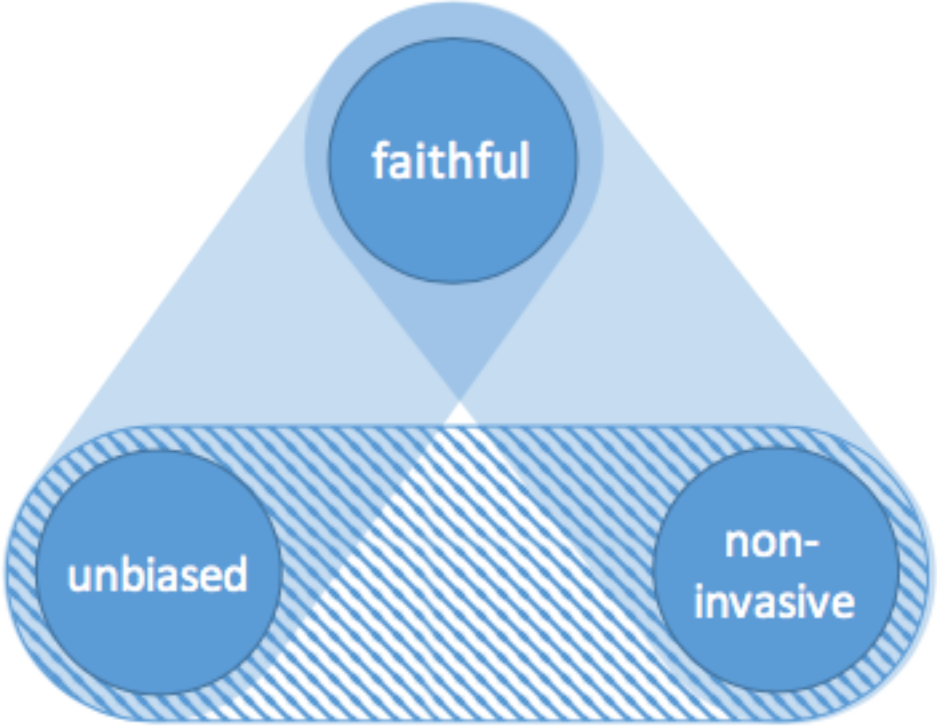
Example

$$|00\rangle\,\langle 00|$$

Ideal measurements

An **ideal** measurement satisfies **3** fundamental properties.

1. **Unbiased**
2. **Faithful**
3. **Non-invasive**



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1. **Unbiased:** The prob. of finding the pointer in the state "i" after the interaction is the same as the prob. of finding the system in $|i\rangle$ before the interaction.

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$\tilde{\rho}_{SP}$ has perfect correlation: on observing the pointer outcome "i" one concludes, with certainty, that the system is left in the corresp. state "i".

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3. **Non-invasive:** The probability of finding the system in the state | i ⟩ is the same before and after the interaction with the pointer,

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Ideal measurements: an example

An ideal measurement of a qubit system by a 1-qubit pointer.

$$\rho_S \otimes \rho_P \longrightarrow \tilde{\rho}_{SP}$$

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$$\begin{pmatrix} a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & b & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & b \end{pmatrix}$$

$$\rho_S = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \qquad \tilde{\rho}_S = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$$

Ideal measurements: an example

An ideal measurement of *any* dimensional system by a 1-qubit pointer.

$$\tilde{\rho}_{SP} = U_{\mathrm{CNOT}} \left(\rho_S \otimes |0\rangle\langle 0|_P\right) U^\dagger_{\mathrm{CNOT}}$$

1. **Unbiased**
2. **Faithful**
3. **Non-invasive**

$$U_{CNOT} := |0\rangle\langle 0|_S \otimes \mathbb{1}_P + \sum_{i\neq 0} |i\rangle\langle i|_S \otimes X_P^{(i)}$$

$$X_P^{(i)} = |0\rangle\langle i| + |i\rangle\langle 0| + \sum_{j\neq 0,i} |j\rangle\langle j|$$

Ideal measurements

A measurement is non-ideal if any one of the **3** properties fails to hold.

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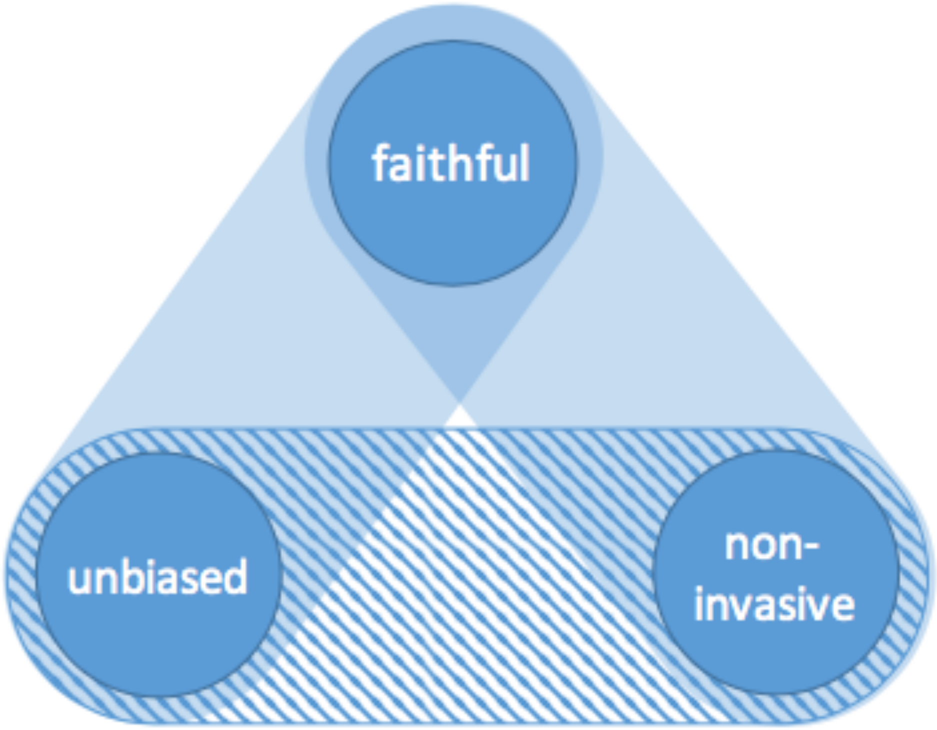

Theorem: ideal measurements are not possible

- **Faithful** measurements (perfect correlations) are possible **if and only if** one can prepare pure states.

- By the third law of thermodynamics, one cannot prepare pure states, therefore **faithful** measurements not possible → ideal measurements are not physically feasible.

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Creating pure states costs infinite energy, takes infinite time, or requires infinite particles.

L. Masanes and J. Oppenheim, "A general derivation and quantification of the third law of thermodynamics," Nat. Commun. 8, 14538 (2017), arXiv:1412.3828.
L. J. Schulman, T. Mor, and Y. Weinstein, "Physical Limits of Heat-Bath Algorithmic Cooling," Phys. Rev. Lett. 94, 120501 (2005). H. Wilming and R. Gallego, "Third Law of Thermody- namics as a Single Inequality," Phys Rev. X 7, 041033 (2017), arXiv:1701.07478.
F. Clivaz, R. Silva, G. Haack, J. Bohr Brask, N. Brun- ner, and M. Huber, "Unifying paradigms of quan- tum refrigeration: resource-dependent limits," (2017), arXiv:1710.11624.
J. Scharlau and M. P. Mu¨ller, "Quantum Horn's lemma, finite heat baths, and the third law of thermodynamics," Quantum 2, 54 (2018), arXiv:1605.06092.

Approximating ideal measurements

Since ideal measurements are not possible, we would like to determine how closely they can be approximated.

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2. **Faithful** 
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Faithful means the pointer never shows the wrong outcome

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$$\rho_P = \tau_P(\beta) = \frac{1}{\mathcal{Z}} \begin{pmatrix} 1 & 0 \\ 0 & e^{-\beta E_P} \end{pmatrix}$$

$$\beta = \frac{1}{T}$$

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Before: $\rho_S \otimes \rho_P \longrightarrow \tilde{\rho}_{SP}$

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U is unbiased (preserves the probabilities)

Energy cost of unbiased measurements

Goal: to bring $\tilde{\rho}_{SP}$ as close as possible to the ideal post-interaction state. We know that pure states make the best pointers.

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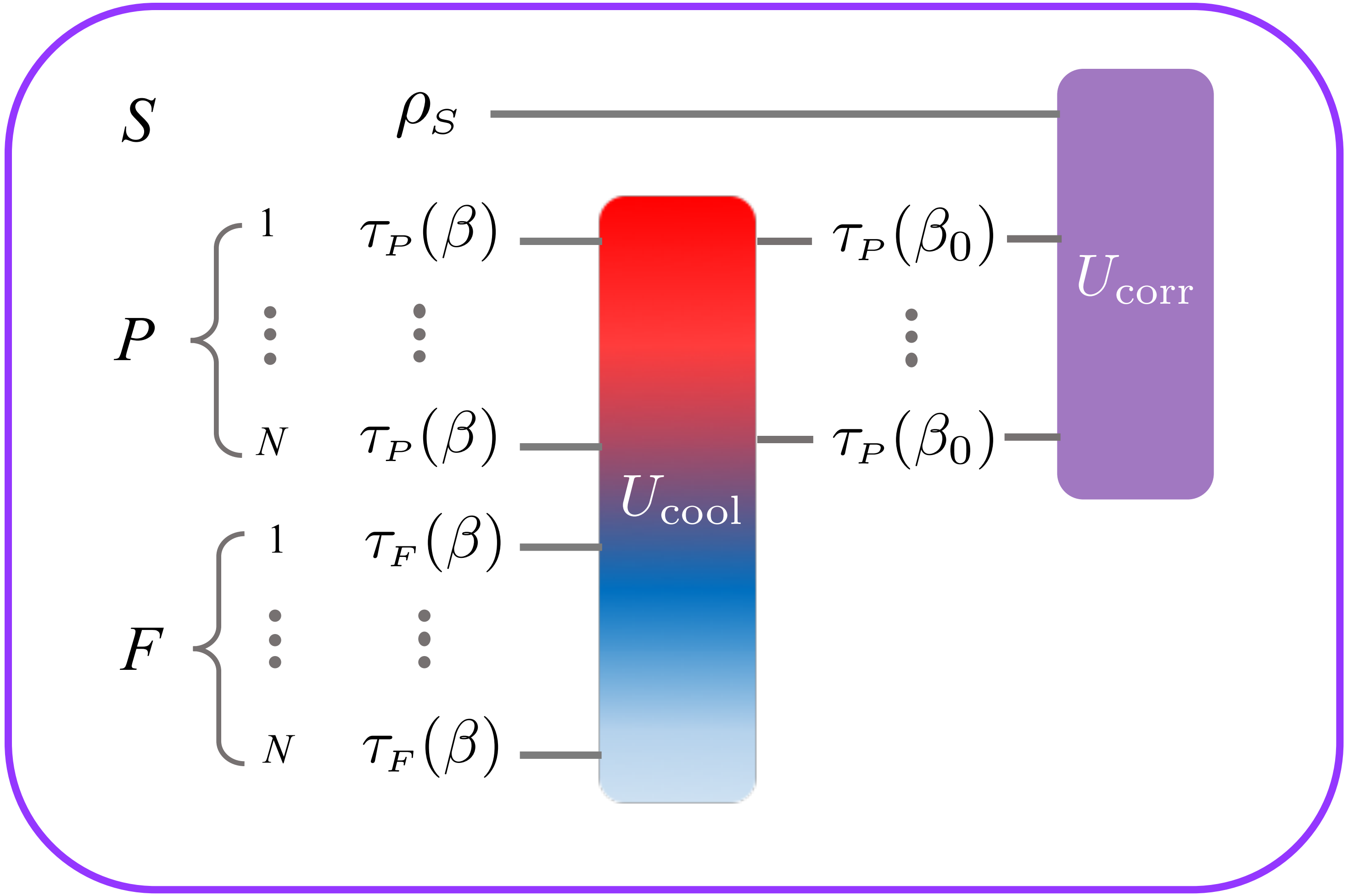
1. **Cool** the pointer (bring it closer to a pure state).

2. **Correlate** the pointer and system (generate as much correlation as possible to effect the measurement).

Schematic of setup



Energy cost of unbiased measurements

$$\Delta E = \boxed{\Delta E_{\mathrm{cool}}} + \boxed{\Delta E_{\mathrm{corr}}}$$

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1. **Cool** the pointer. $\beta \longrightarrow \beta_0$

2. **Correlate** the pointer and system apply by applying the unitary that achieves the best possible correlation for the new temperature and number of particles at the lowest energy cost

$$\min_{U_{\mathrm{corr}}} \Delta E_{\mathrm{corr}} \quad \text{s.t.} \quad C(\tilde{\rho}_{SP}) = C_{\max}(\beta_0, N)$$

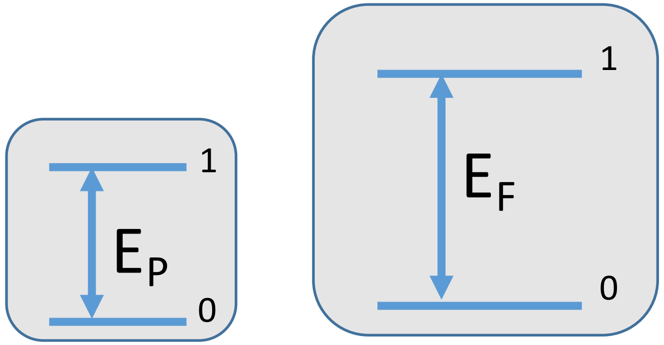
analytic construction

Energy cost of unbiased measurements

$$\Delta E = \boxed{\Delta E_{\text{cool}}} + \boxed{\Delta E_{\text{corr}}}$$

1. **Cool** the pointer. **1-qubit fridge** $\tau_P(\beta)^{\otimes N} \mapsto \tau_P(\beta \frac{E_F}{E_P})^{\otimes N}$

$$\Delta E_{\text{cool}} = N(E_F - 1)\left(\frac{1}{e^{-\beta E_F}+1} - \frac{1}{e^{-\beta E_P}+1}\right)$$



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Energy cost of unbiased measurements

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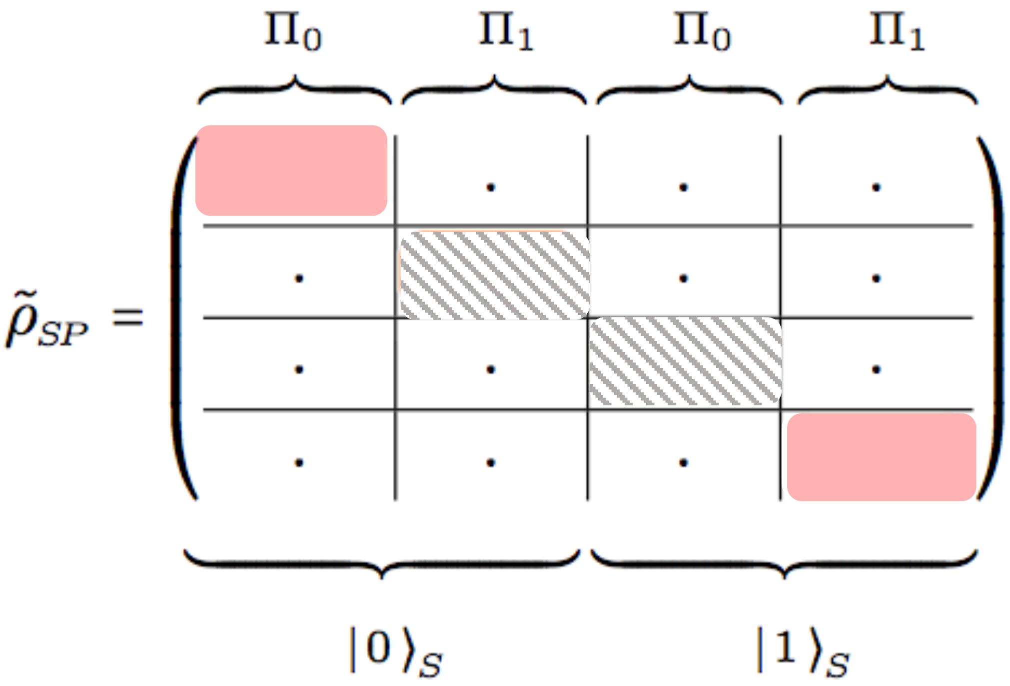
2. **Correlate** the pointer and system.

ΔE_{corr} analytic and correlations:

$$C_{\max}(\beta, N) = \frac{1}{\mathcal{Z}^N} \sum_{k=0}^{N/2} \binom{N}{k} e^{-k E_P \beta} .$$



Best correlations fo N-qubit pointer

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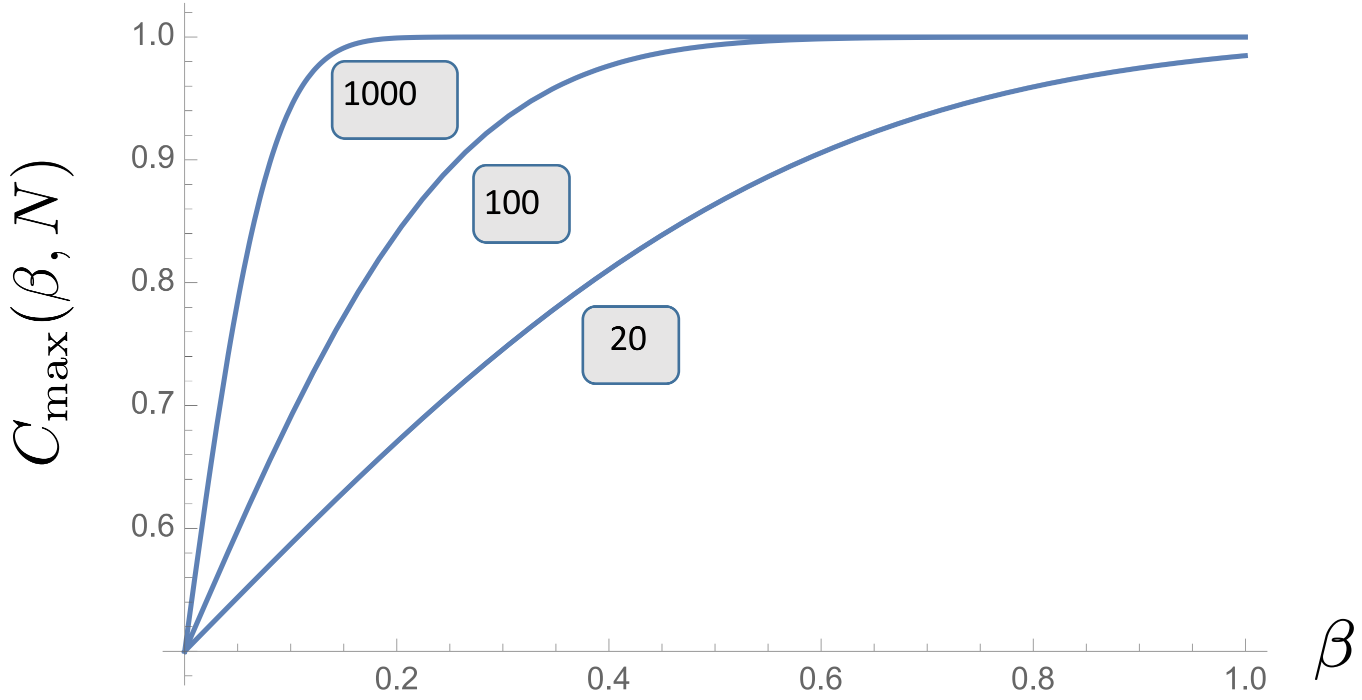
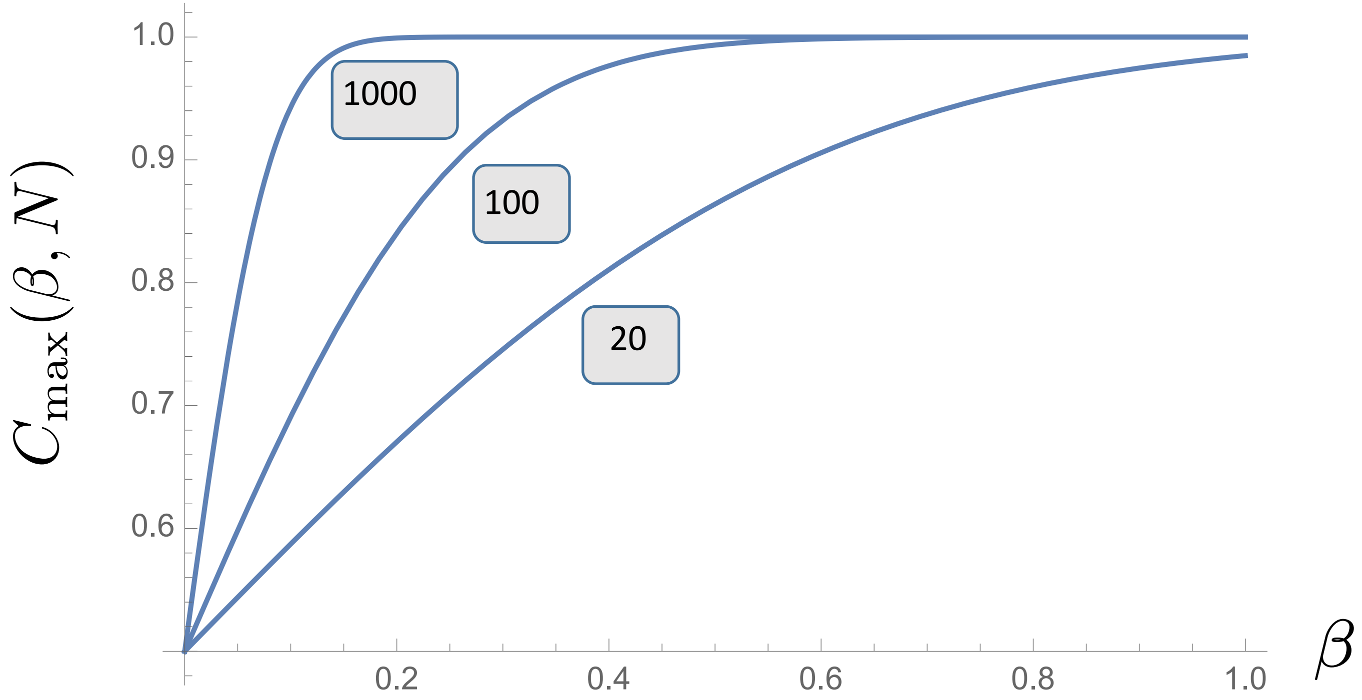


Best correlations fo N-qubit pointer

$$\lim_{\beta\to\infty}\left(C_{\max}(\beta,N) = \frac{1}{\mathcal{Z}^N}\sum_{k=0}^{N/2}\binom{N}{k}e^{-kE_P\beta}\,.\right) = 1$$



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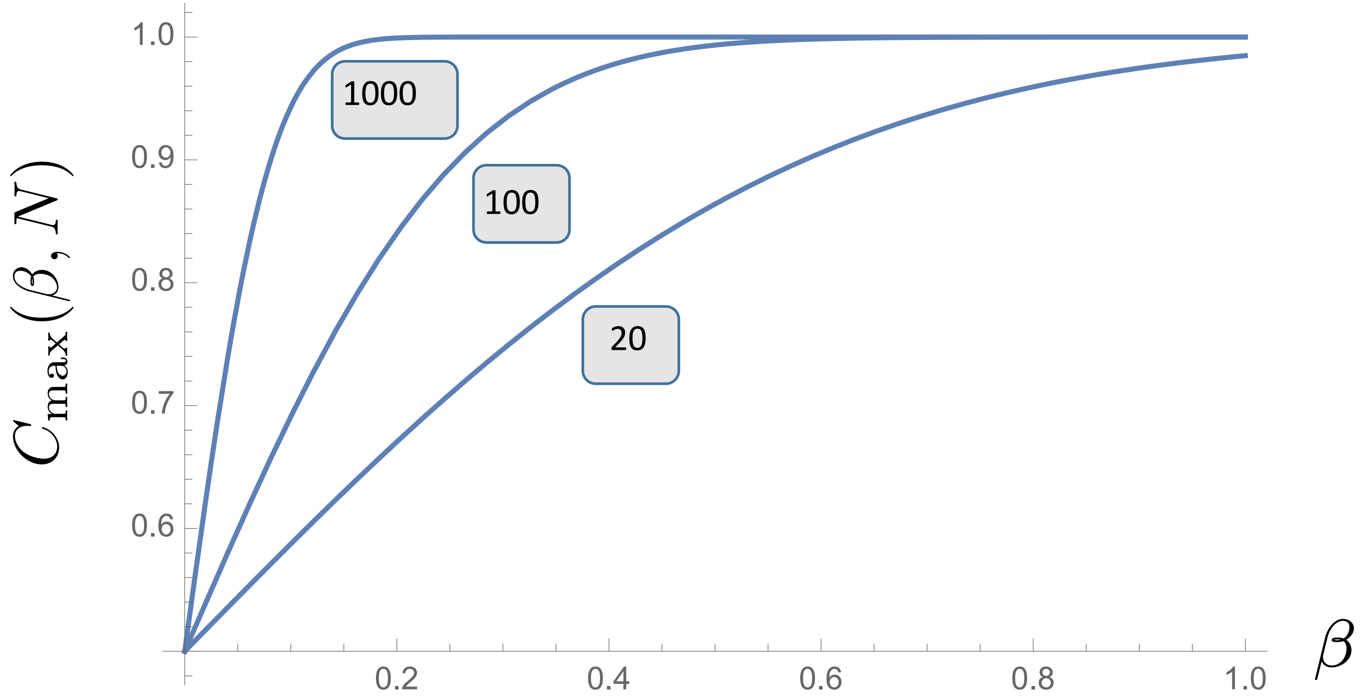
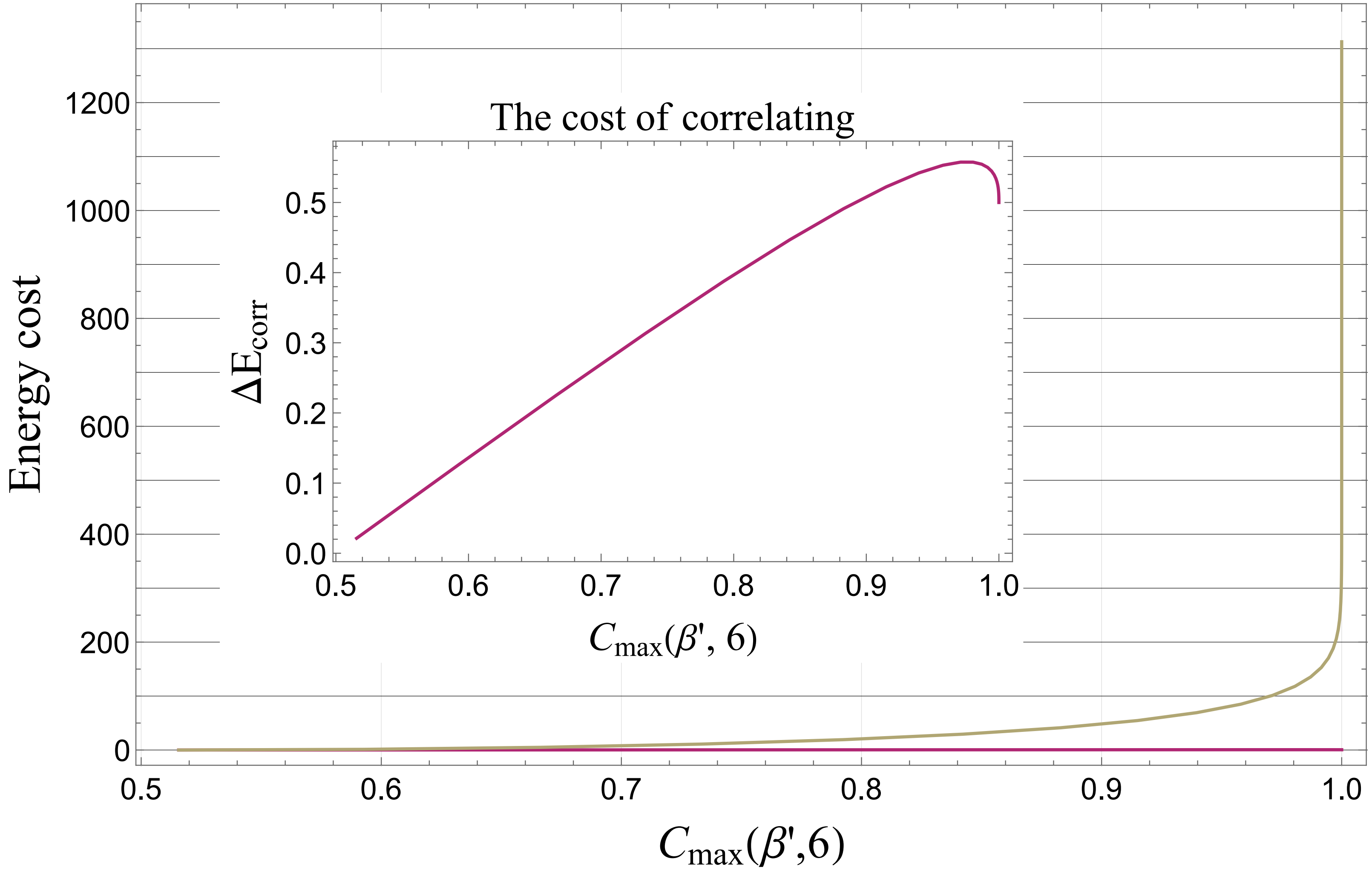


Cost of a non-ideal measurement



- System: 1 qubit
- Pointer: 6-qubits (energy gap in the microwave regime βE_P=1/30)

- Start at room temp and cool with a fridge with gap E_F , until the fridge is "exhausted" .

- Correlate from new temp **β'.**

Summary

- **Theorem:** an unbiased measurement can always be implemented at finite cost and in finite time.
- **Corollary:** to be faithful, the pointer has to be cooled, subject to the laws of thermodynamics.
- **Consequence:** obtaining reliable knowledge about quantum systems generically requires macroscopic quantities of work; perfect knowledge is infinitely expensive.
- **Validity:** results valid for *any* system-pointer Hamiltonian, fundamental bounds phrased in terms of dimension and β.

Future work

- Quantum to classical transition requires macroscopic work → robustness and broadcastability.
- Fundamental 'collapse' and other foundational questions.
- Work Estimation and Work Fluctuations in the Presence of Non-Ideal Measurements (Tiago Debarba, Gonzalo Manzano, Yelena Guryanova, Marcus Huber, Nicolai Friis arXiv:1902.08568)
- Landauer erasure and other schemes using infinite control (or time) trade –off with cost? complexity ← → energy ← → time

Thank you

Group leaders

Yelena Guryanova

Ämin Baumeler

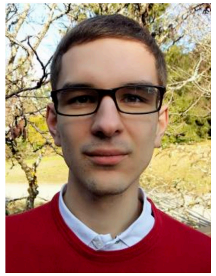
Costantino Budroni

Postdocs

Paul Erker

Simon Milz

Student



Lucien Jezequel

YIRG