Work fluctuations in quasistatic processes

Matteo Scandi, Harry J. D. Miller, Janet Anders,
Martì Perarnau-Llobet

arXiv:1905.07328

Quasistatic processes

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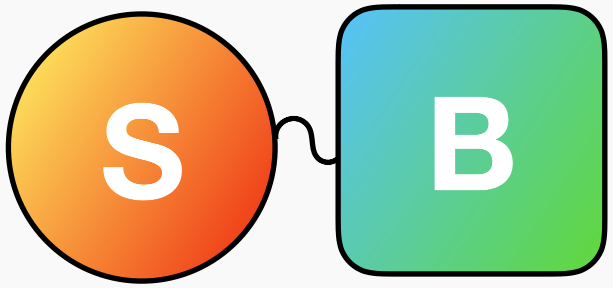
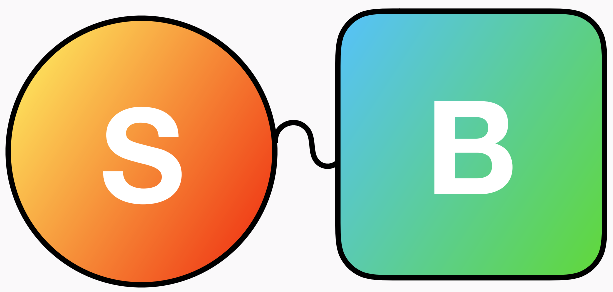


Quasistatic processes

$$H(\{\lambda_t\}) = \sum_i \lambda_t^i X_i$$



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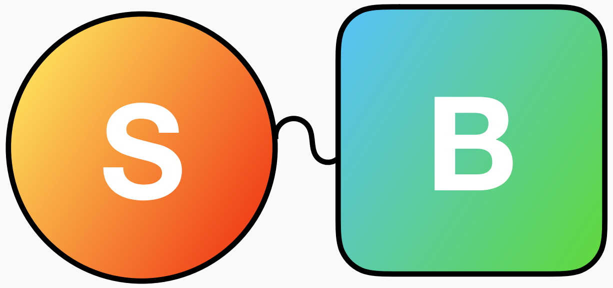
Controllable parameters



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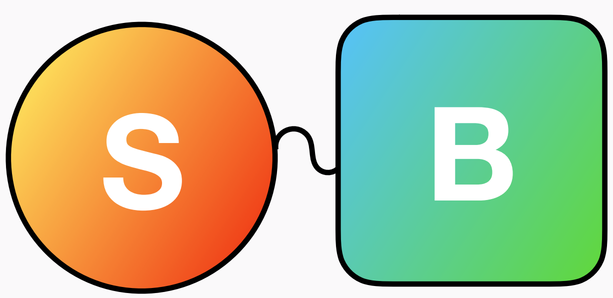

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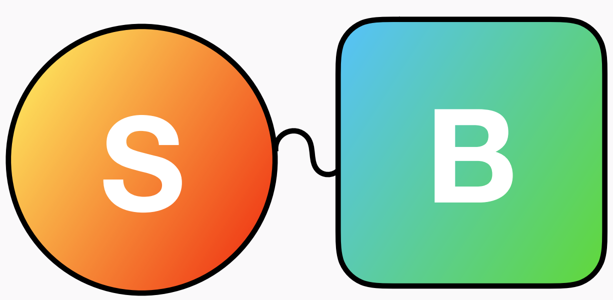

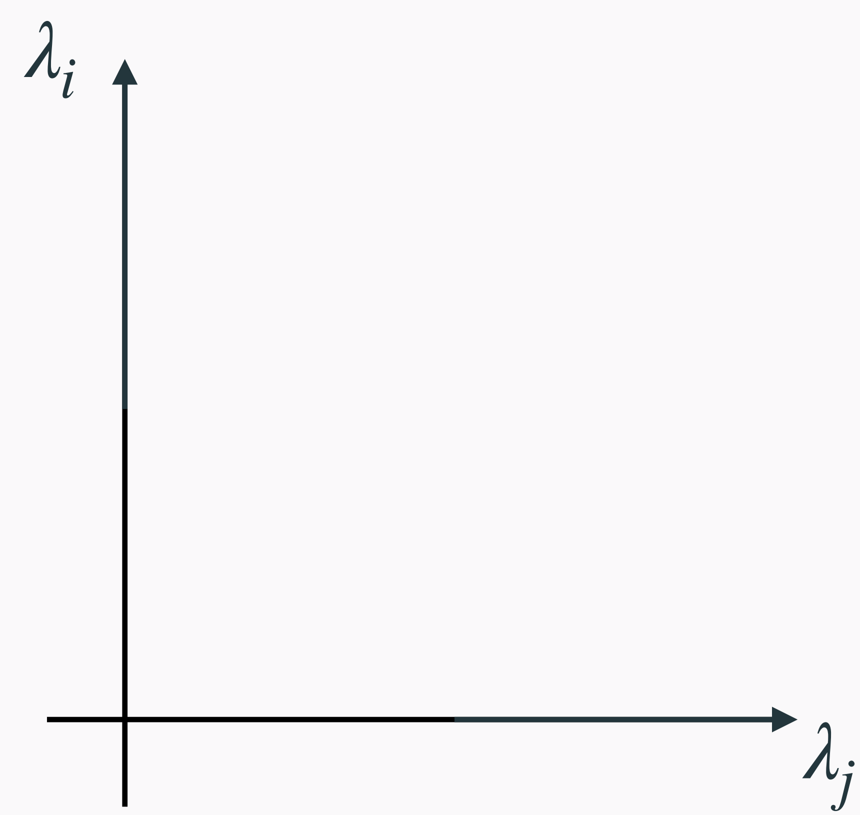

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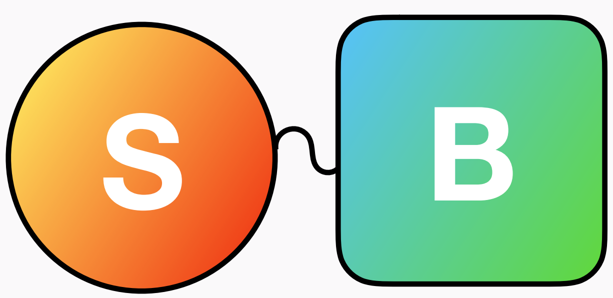

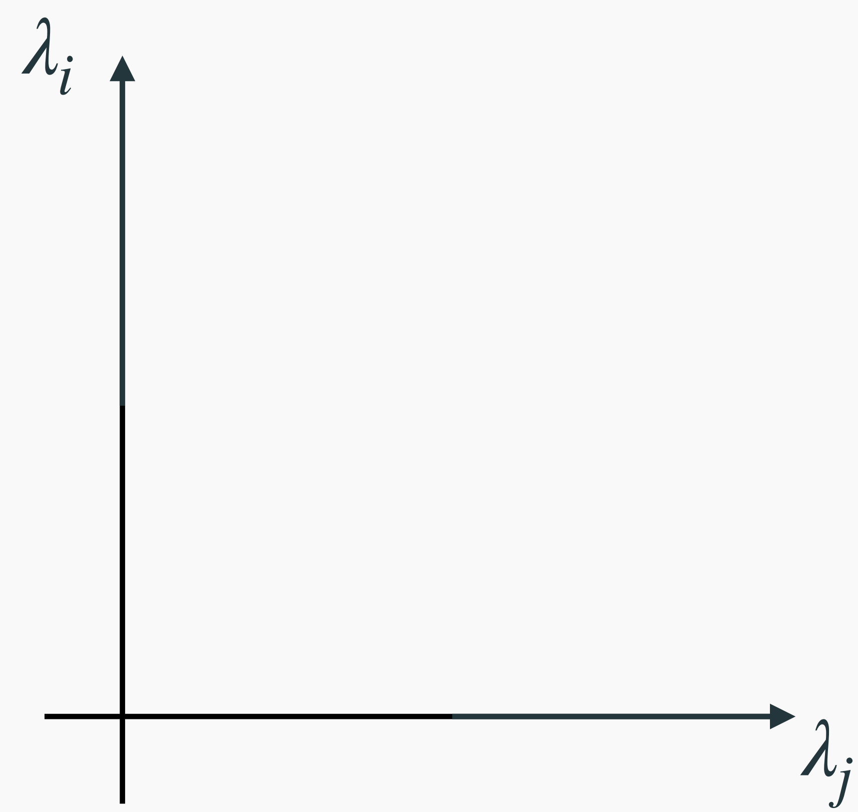

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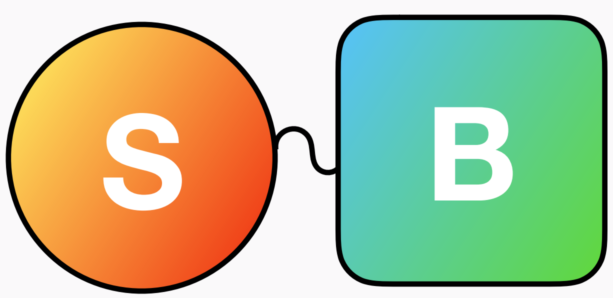

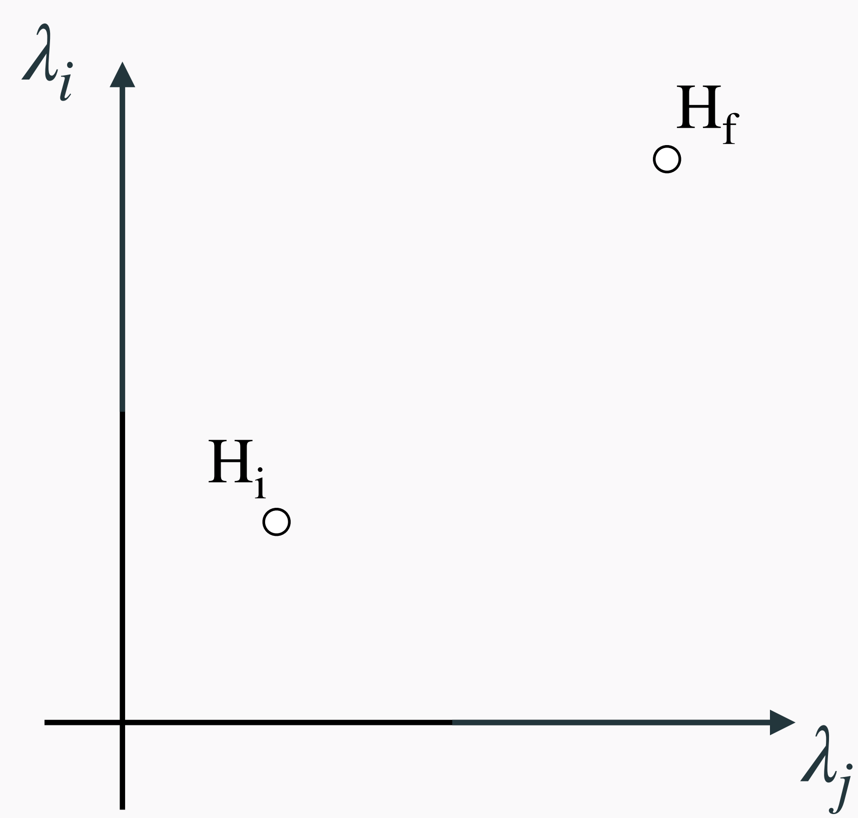

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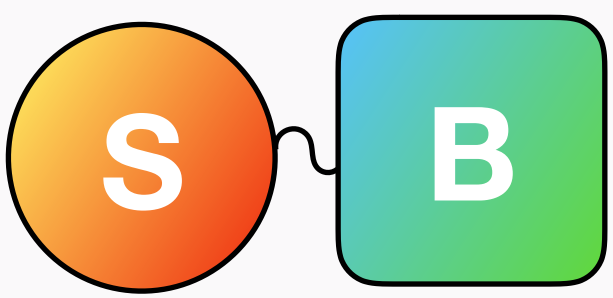

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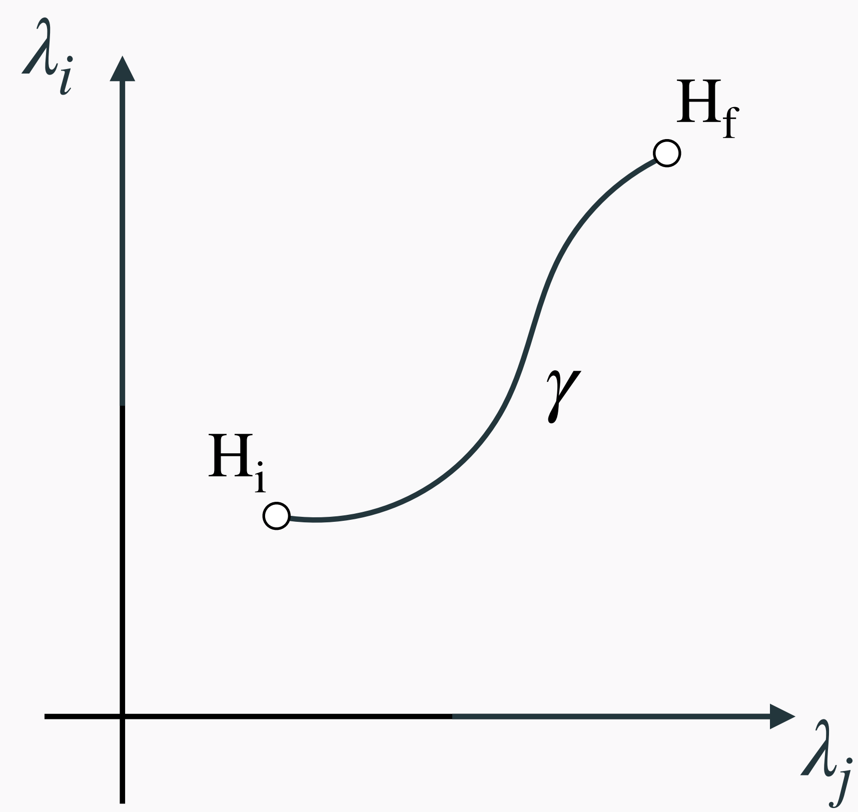

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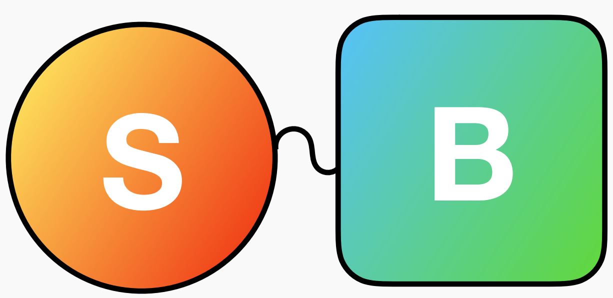

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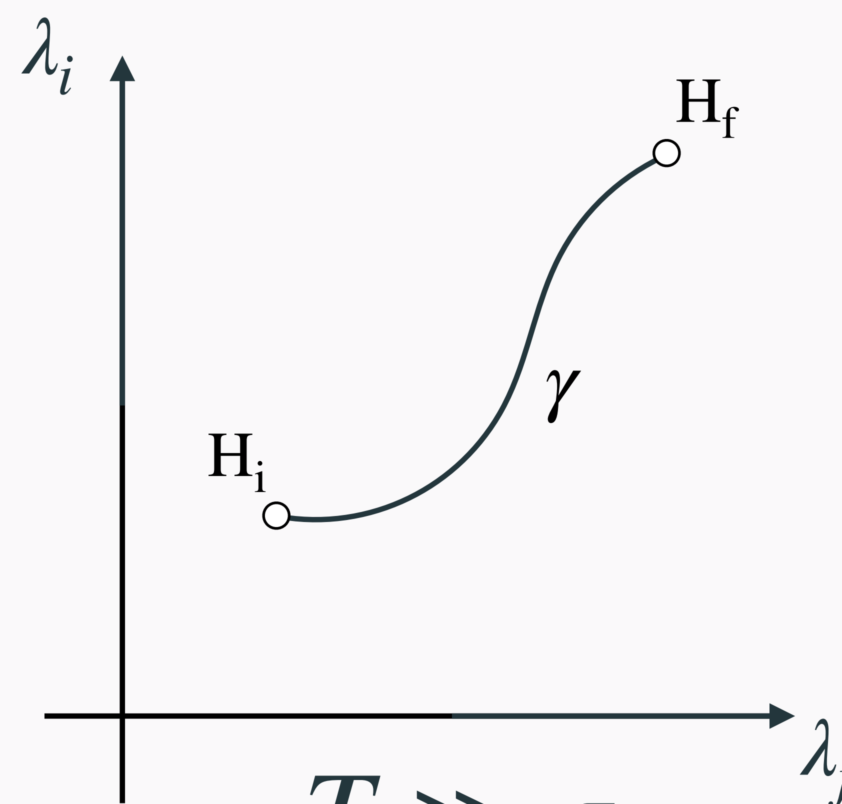

$$T \gg \tau$$

Classical case

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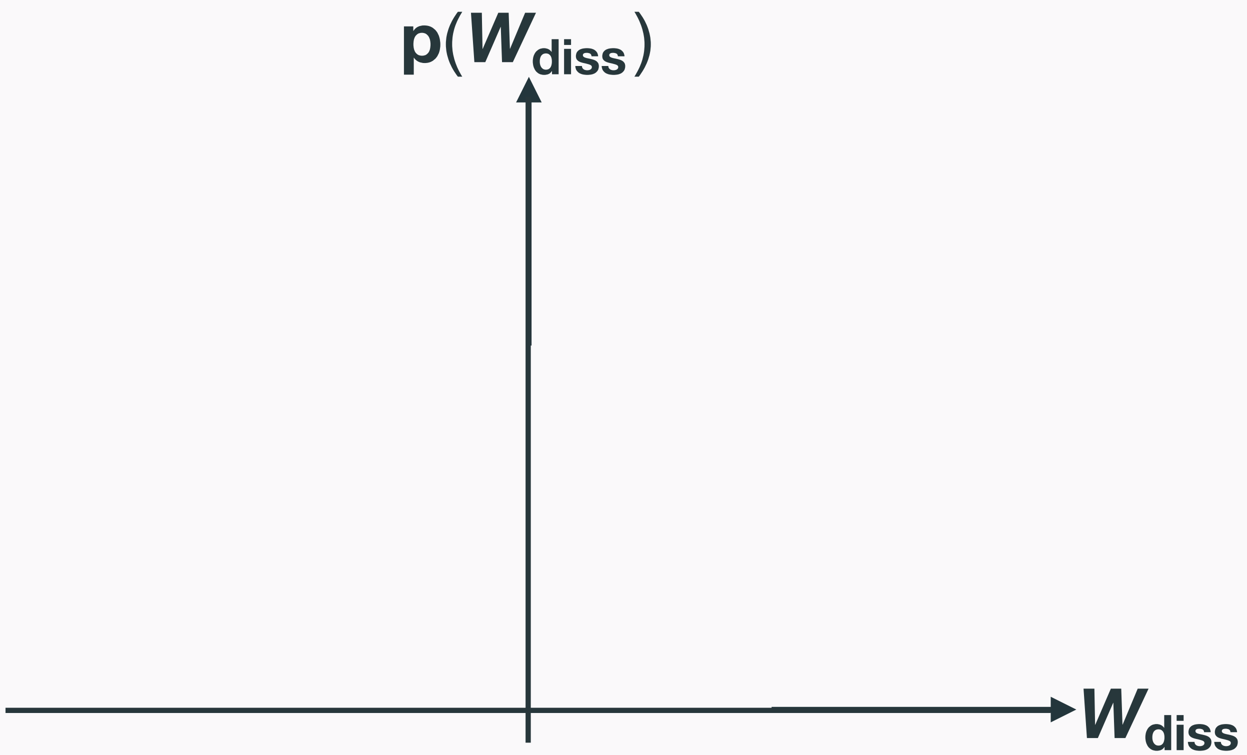

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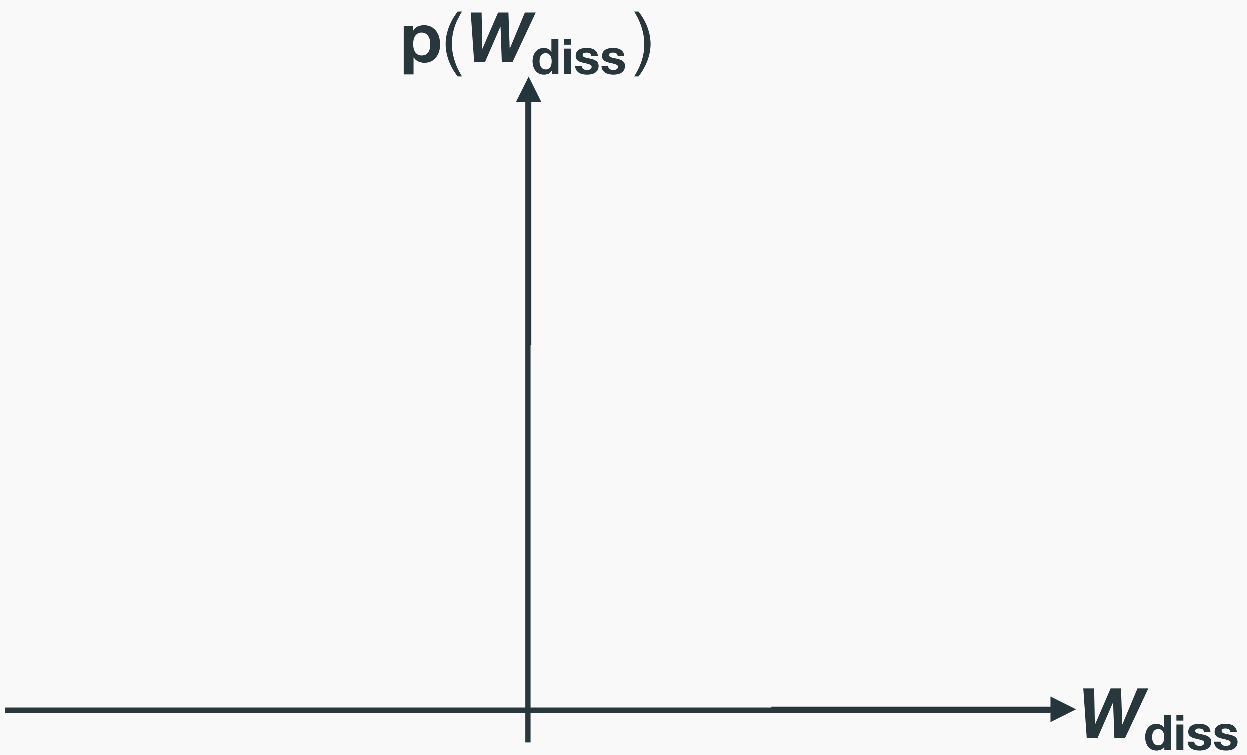

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[1] Specke, Seifert, arXiv:1905.07328

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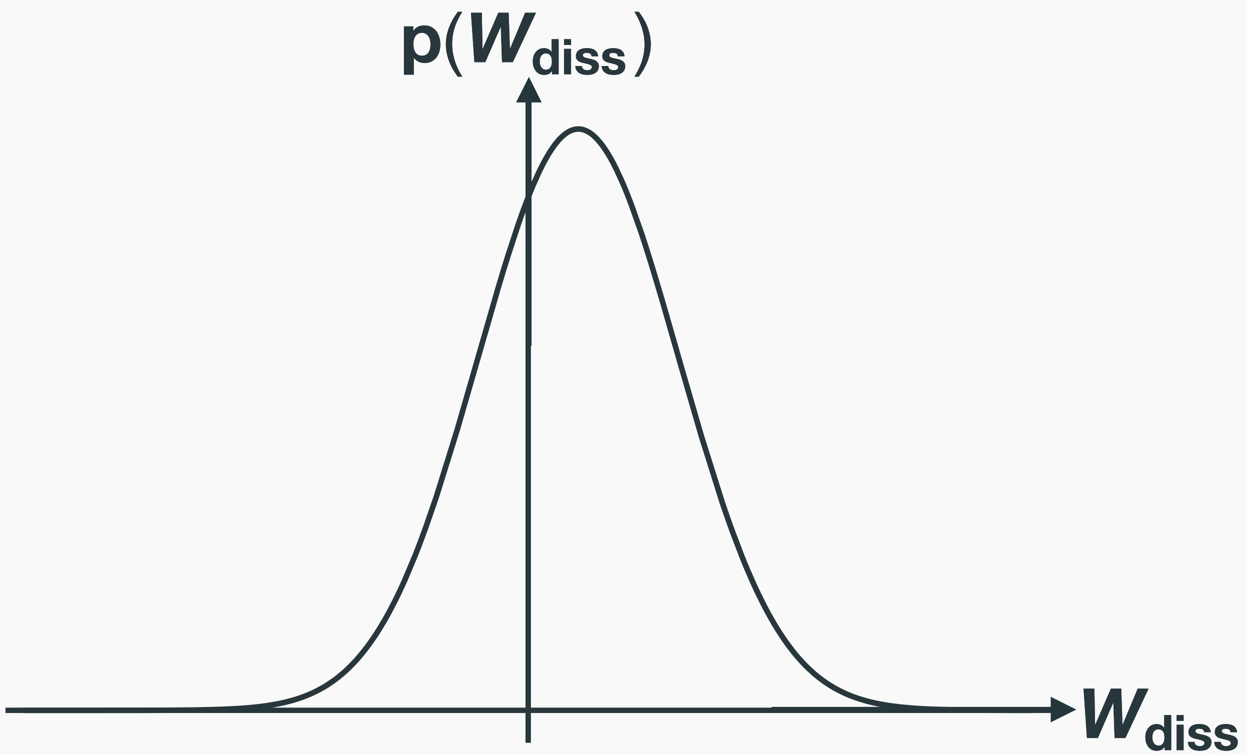

Gaussianity[1]

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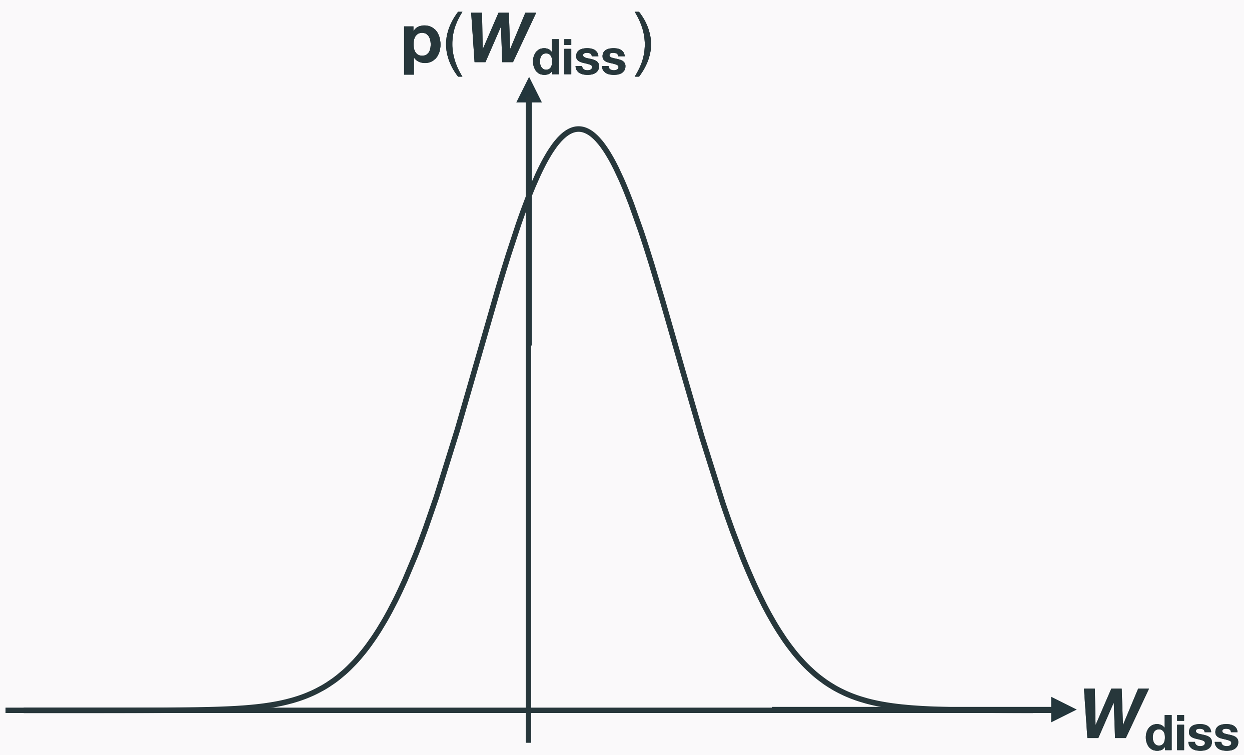

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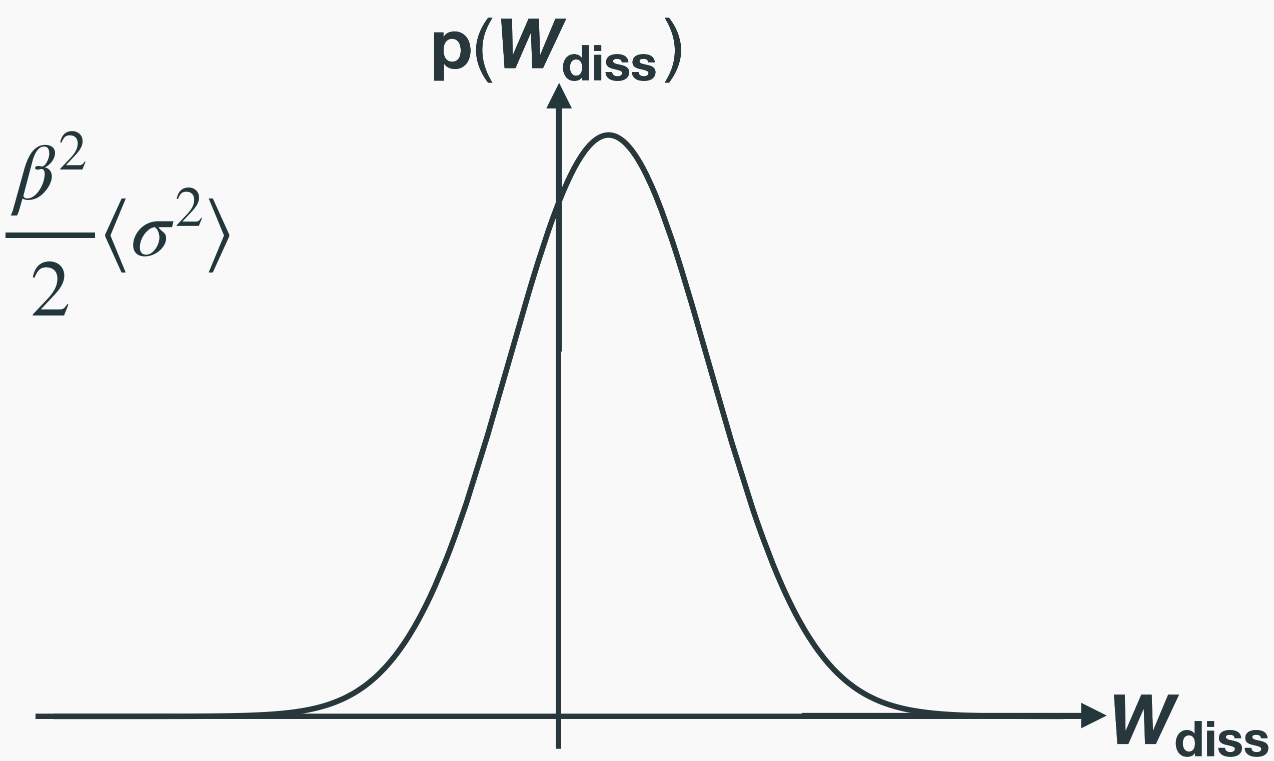

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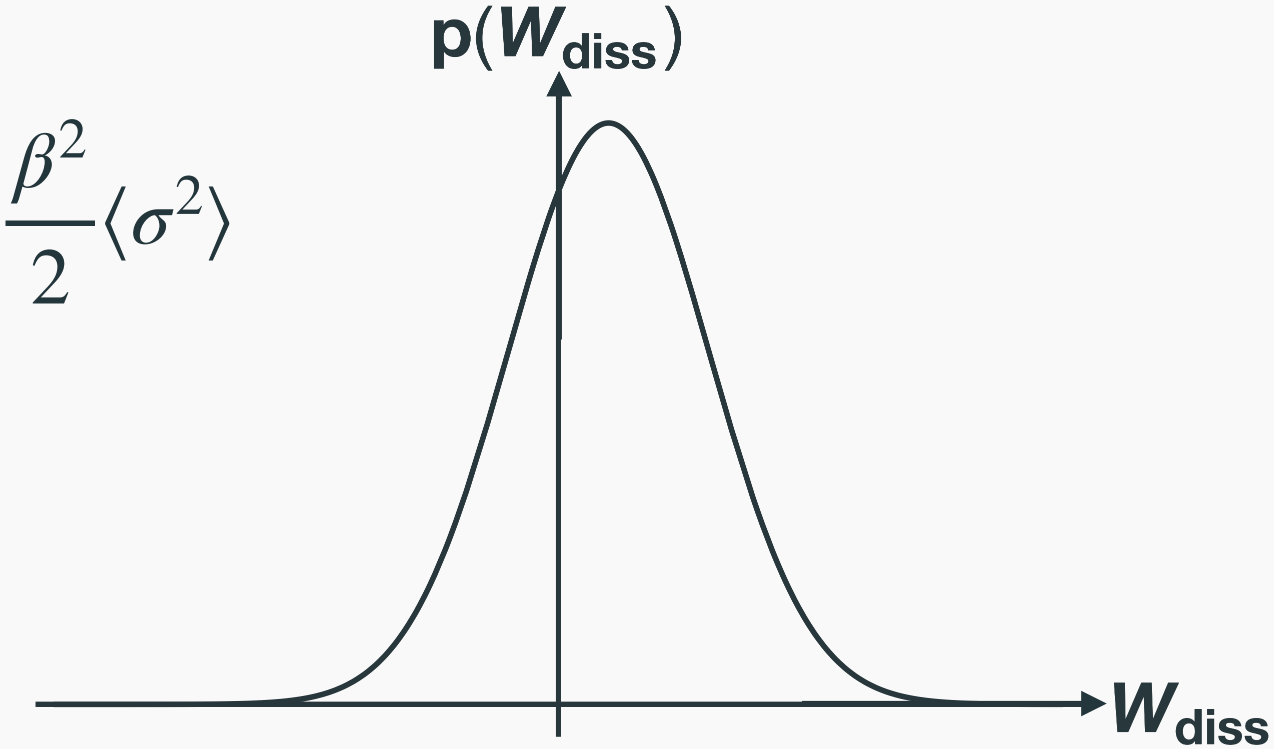

Gaussianity[1]

Fluctuations
Dissipation
Relation

$$\langle W_{diss} \rangle = \frac{\beta}{2} \langle \sigma^2 \rangle$$

Quantum work distribution

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Moment generating function

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$$\langle e^{-\beta\lambda W}\rangle = \mathrm{Tr}[e^{-\beta\lambda H_T}\, U_T\, e^{\beta\lambda H_0}\hat{\rho}_0\, U_T^\dagger]$$

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System

Dynamics

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$$\dot{\rho}_t = \mathscr{L}_t[\rho_t]$$

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Gibbs state $\pi_t := \pi_\beta(H_t)$

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Assumptions:

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2. **Markovianity**

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3. Detailed balance

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Average dissipation

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Drazin inverse of $\mathscr{L}_t$

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Scandi, Perarnau, arXiv:1905.07328

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$$\Bigg\{$$

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$$\frac{\beta}{2}\langle \sigma^2 \rangle = \langle W_{diss} \rangle \quad \Leftrightarrow$$

No coherence in the protocol

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$\mathcal{Q} > 0$ $\Leftrightarrow$ **Non Gaussian work distribution**

(coherence in the protocol)

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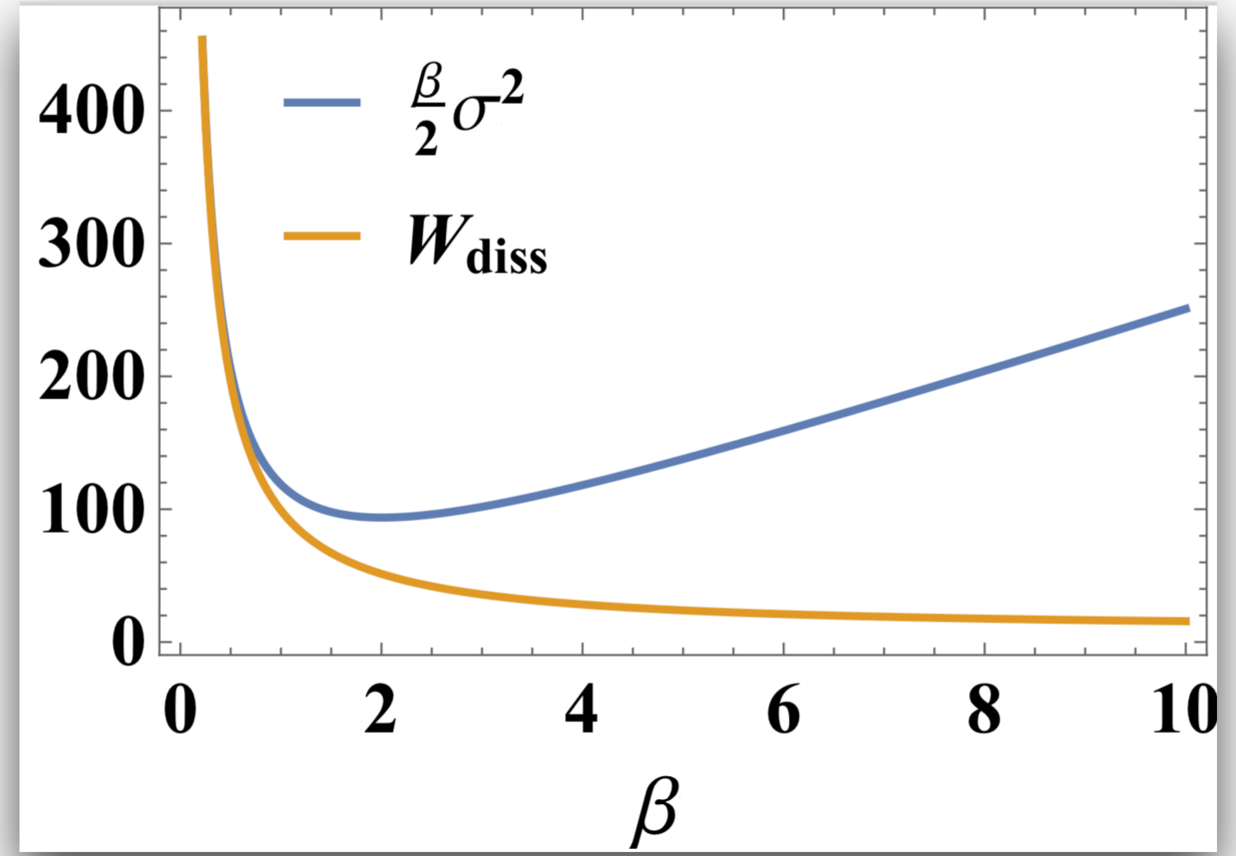

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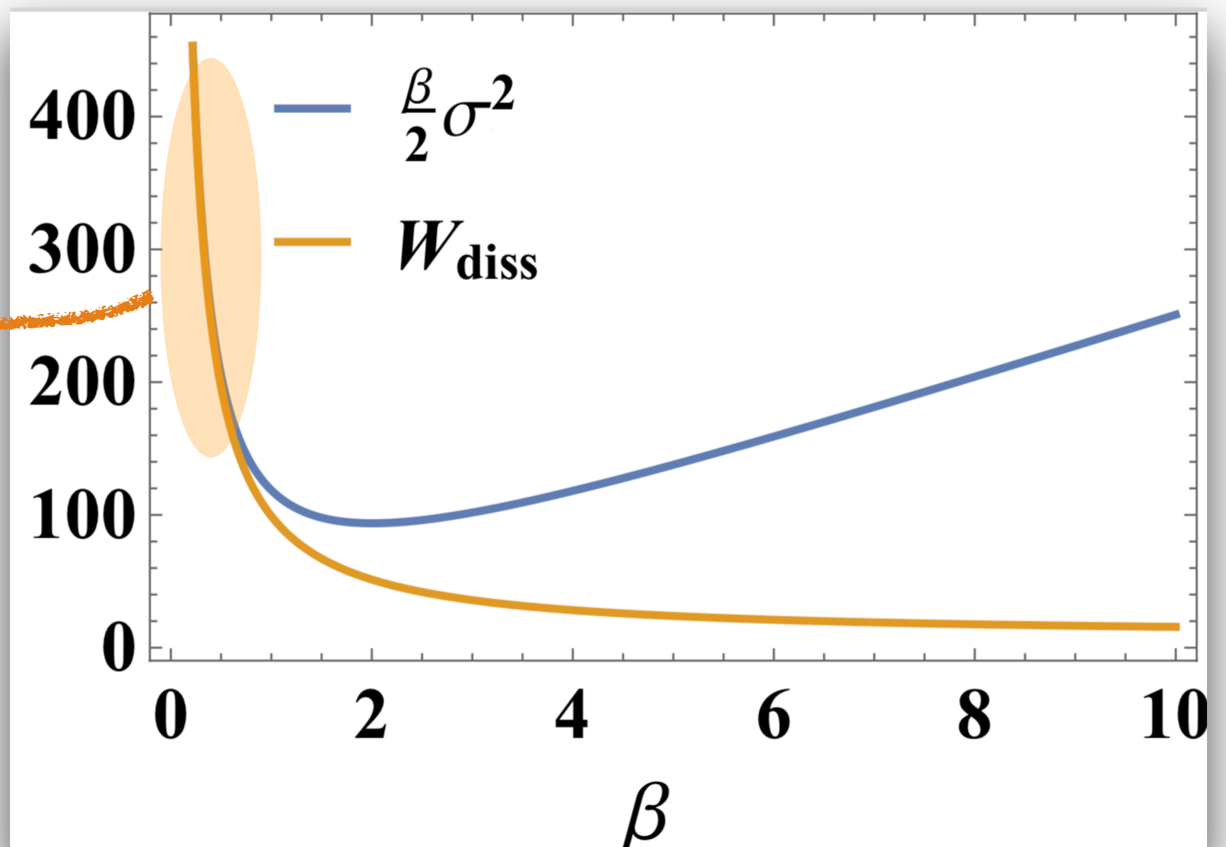

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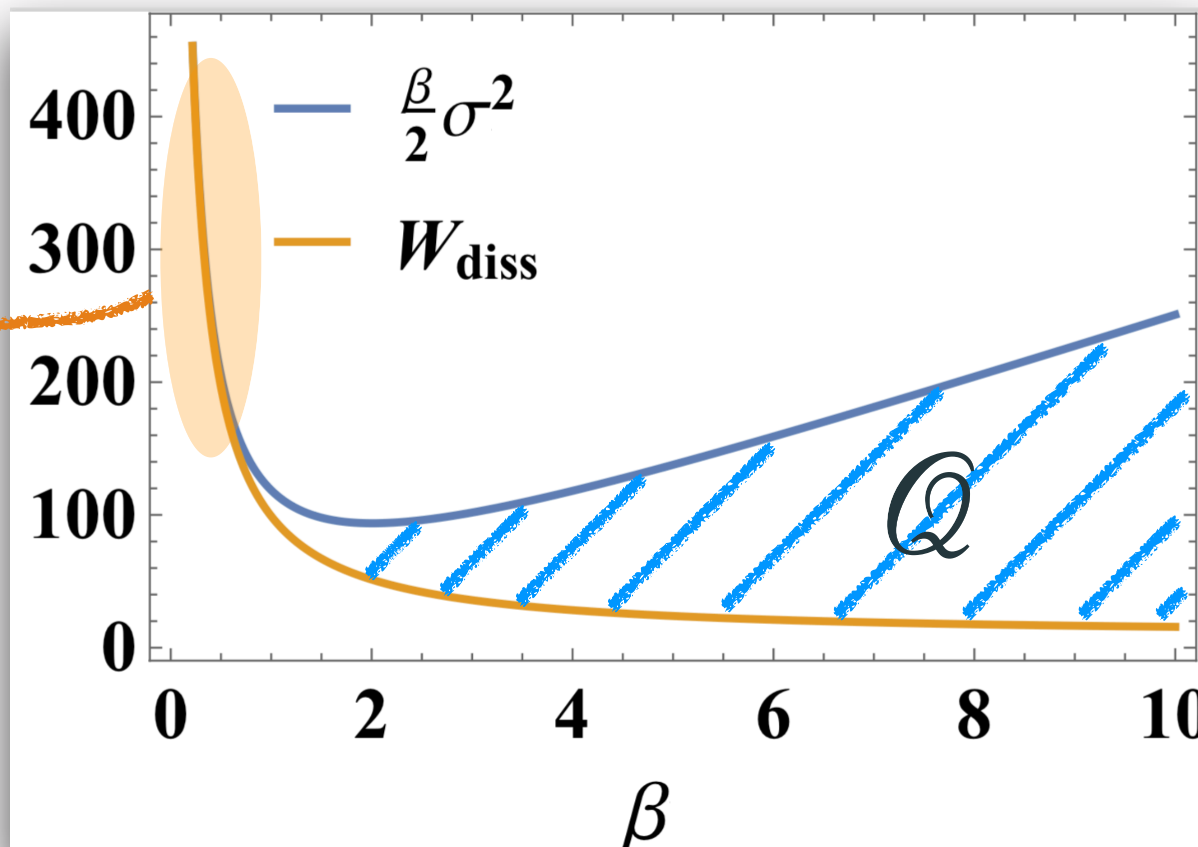

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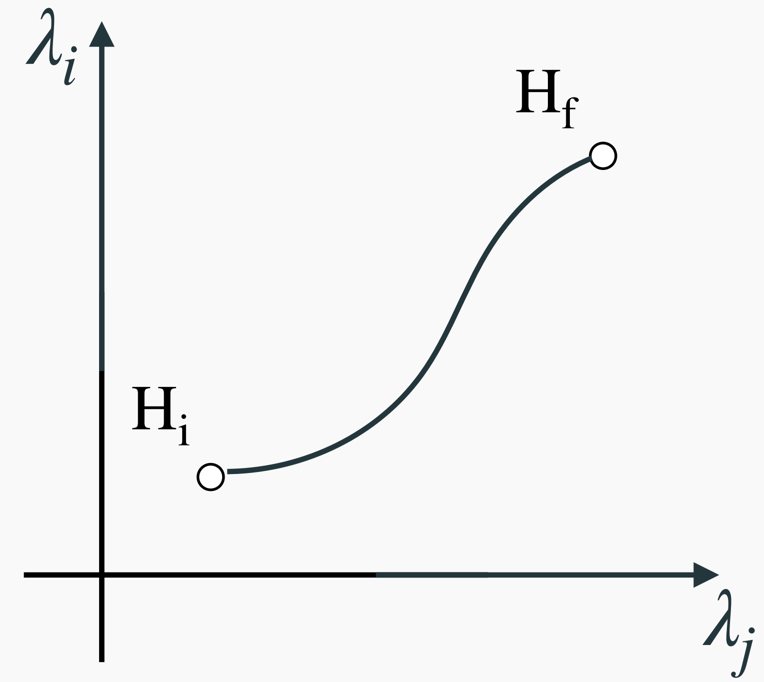

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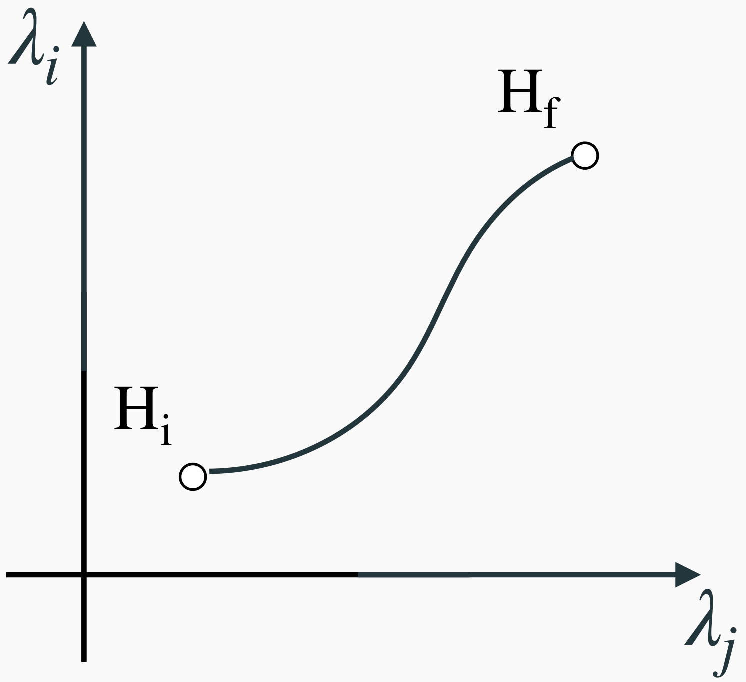

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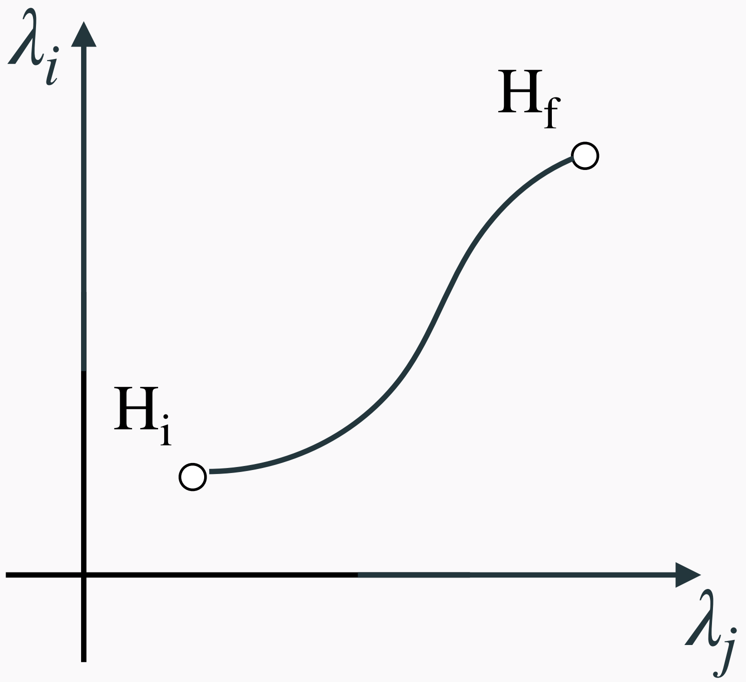

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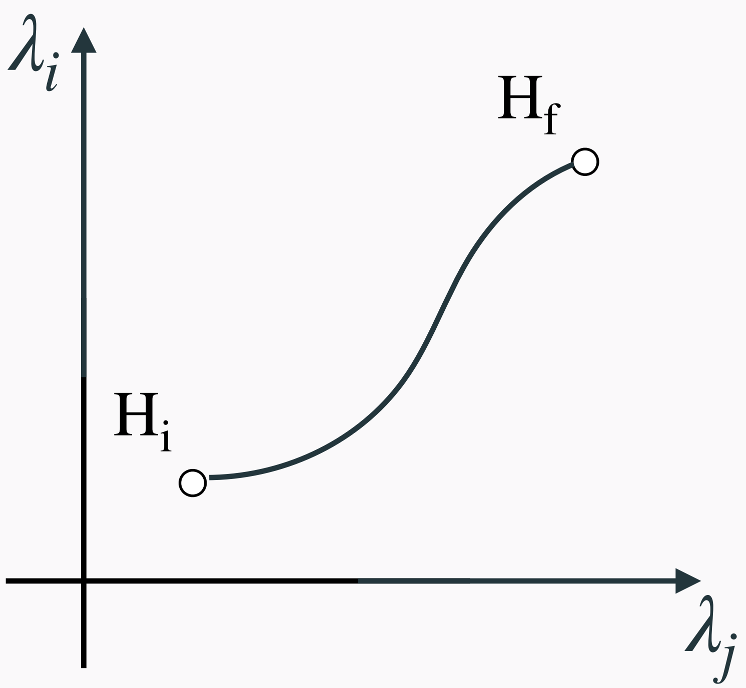

$$\langle W_{diss} \rangle = \frac{1}{T} \int \mathrm{d}t \, \dot{\lambda}_t^i \, \xi_{i,j}(\{\boldsymbol{\lambda_t}\}) \, \dot{\lambda}_t^j$$

$$\frac{\beta}{2} \langle \sigma^2 \rangle = \frac{1}{T} \int \mathrm{d}t \, \dot{\lambda}_t^i \, \Lambda_{i,j}(\{\boldsymbol{\lambda_t}\}) \, \dot{\lambda}_t^j$$

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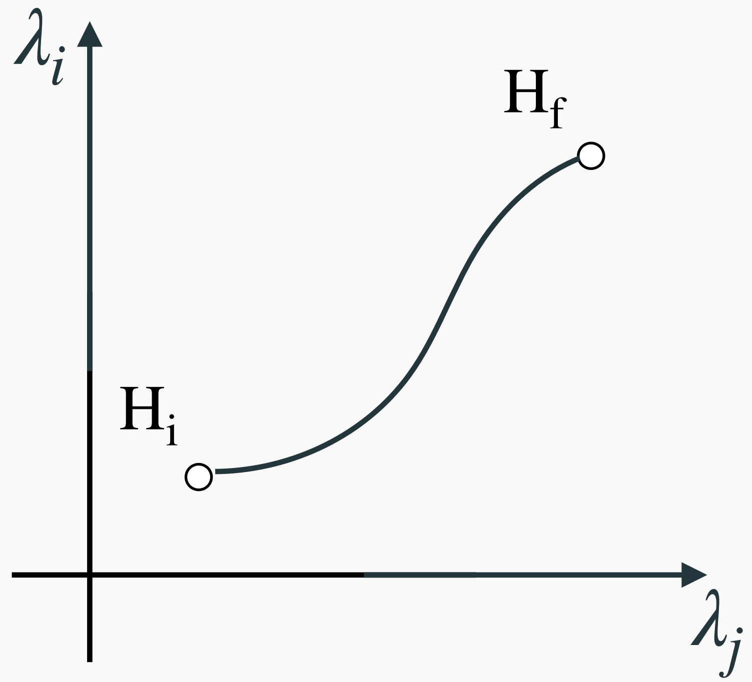

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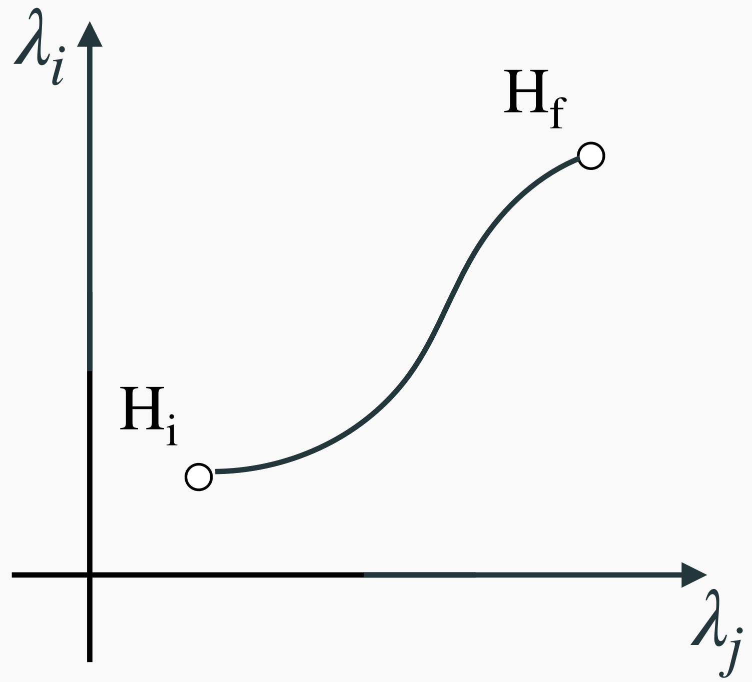

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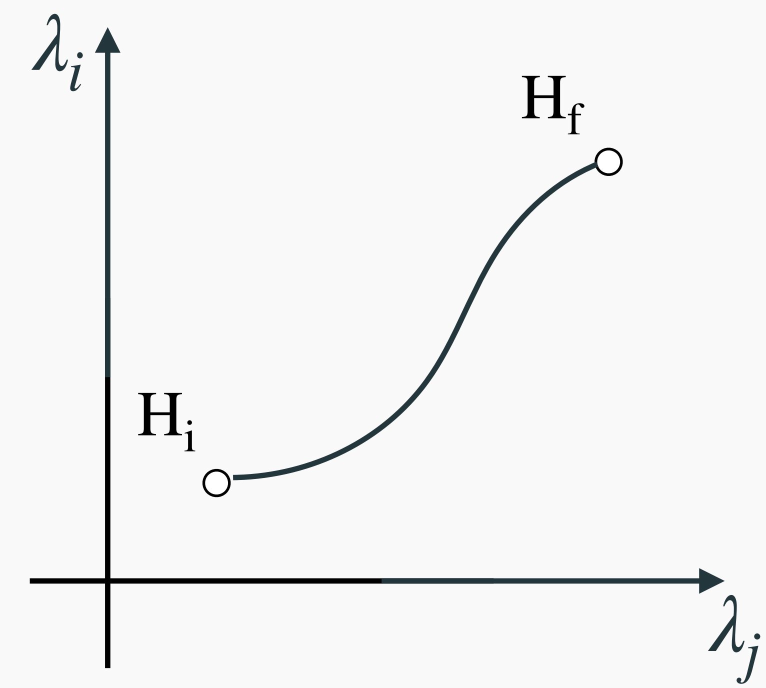
- $\boldsymbol{\Lambda} \geq \boldsymbol{\xi} > 0$

Optimisation protocols



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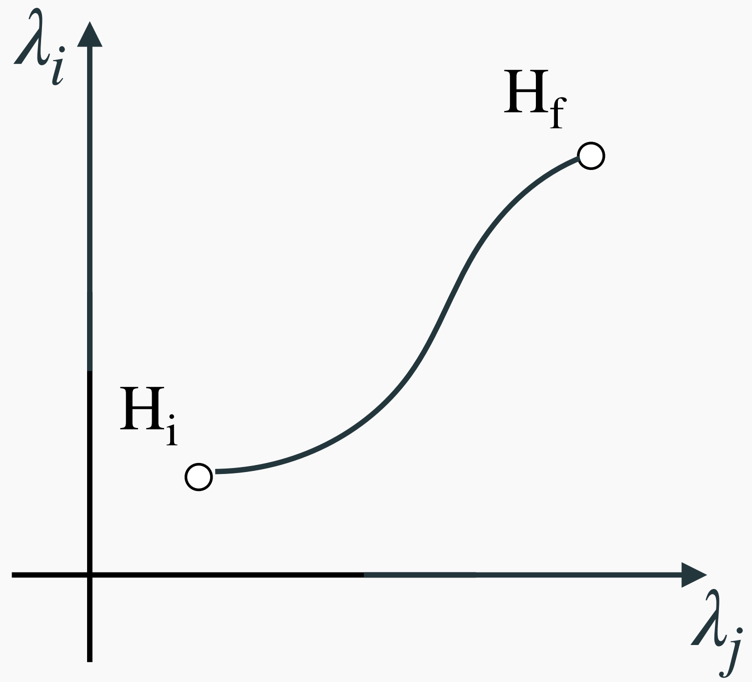

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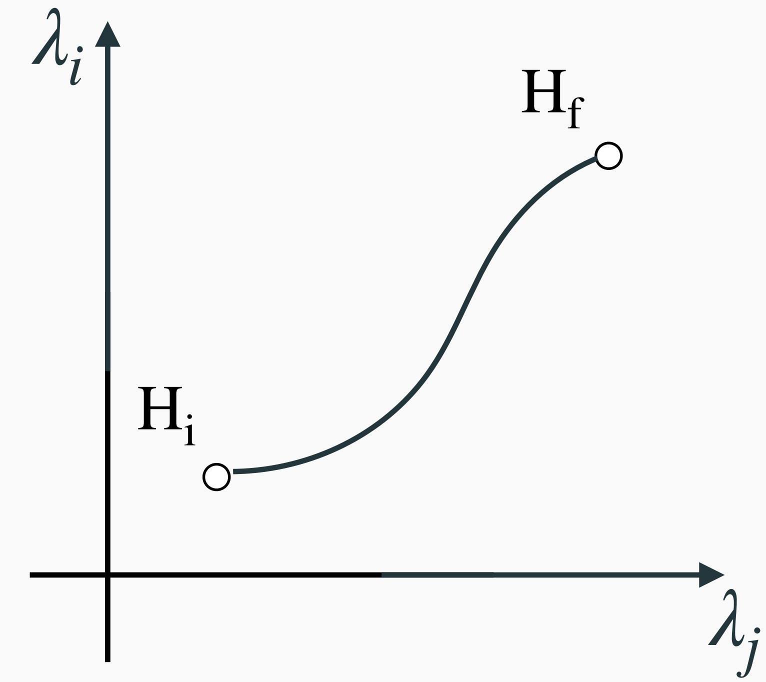

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Metrics

Optimisation protocols

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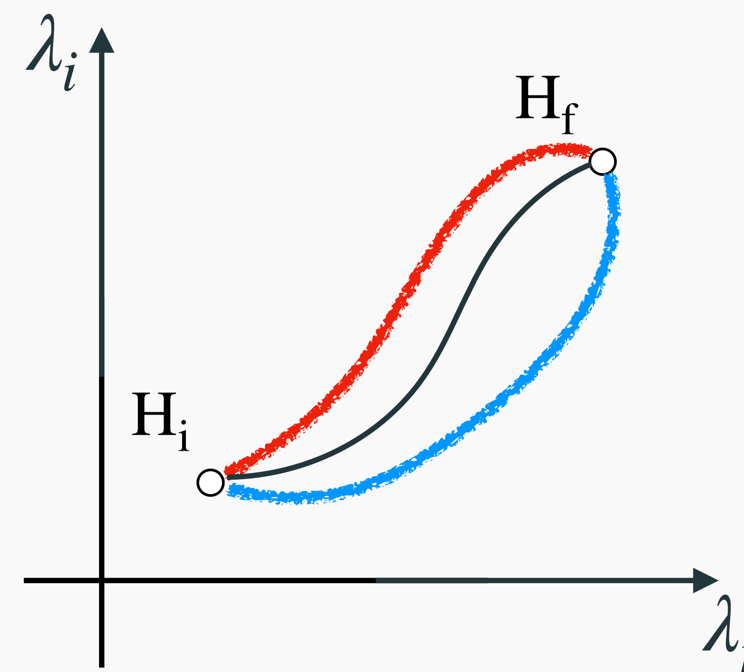

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Metrics

One can find optimal protocols by solving the geodesics equation

Example: Pareto fronts

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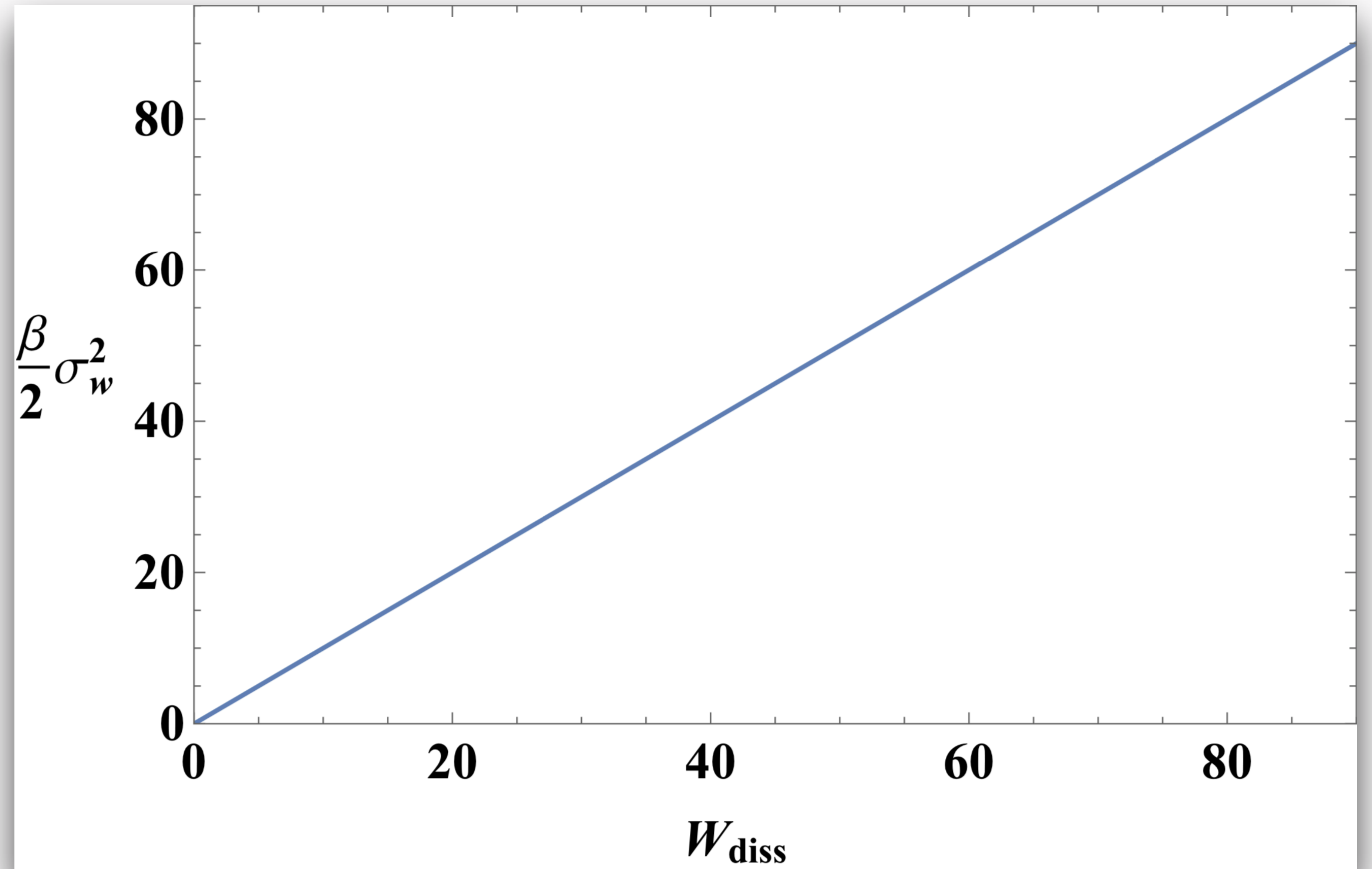
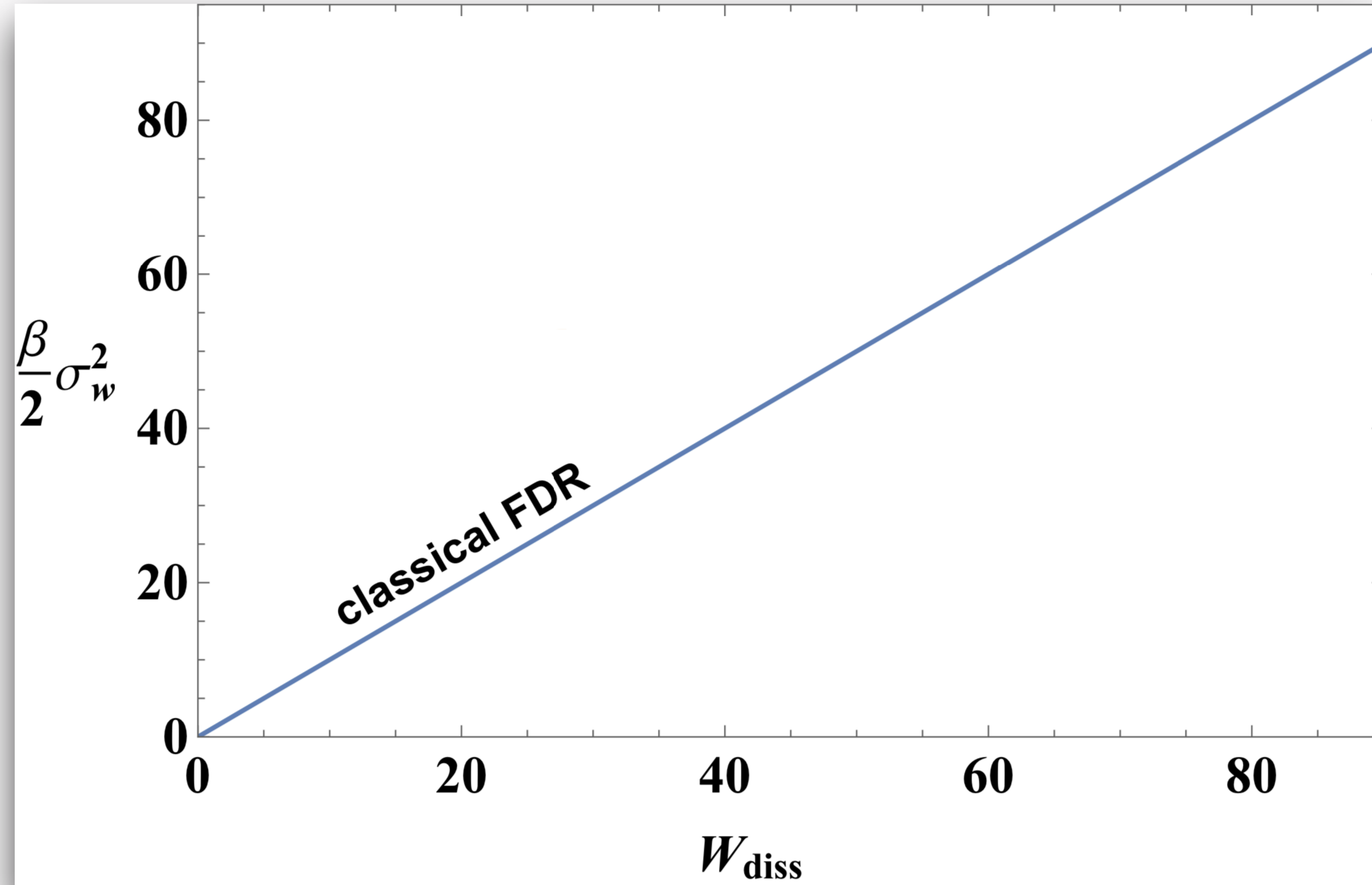


Example: Pareto fronts



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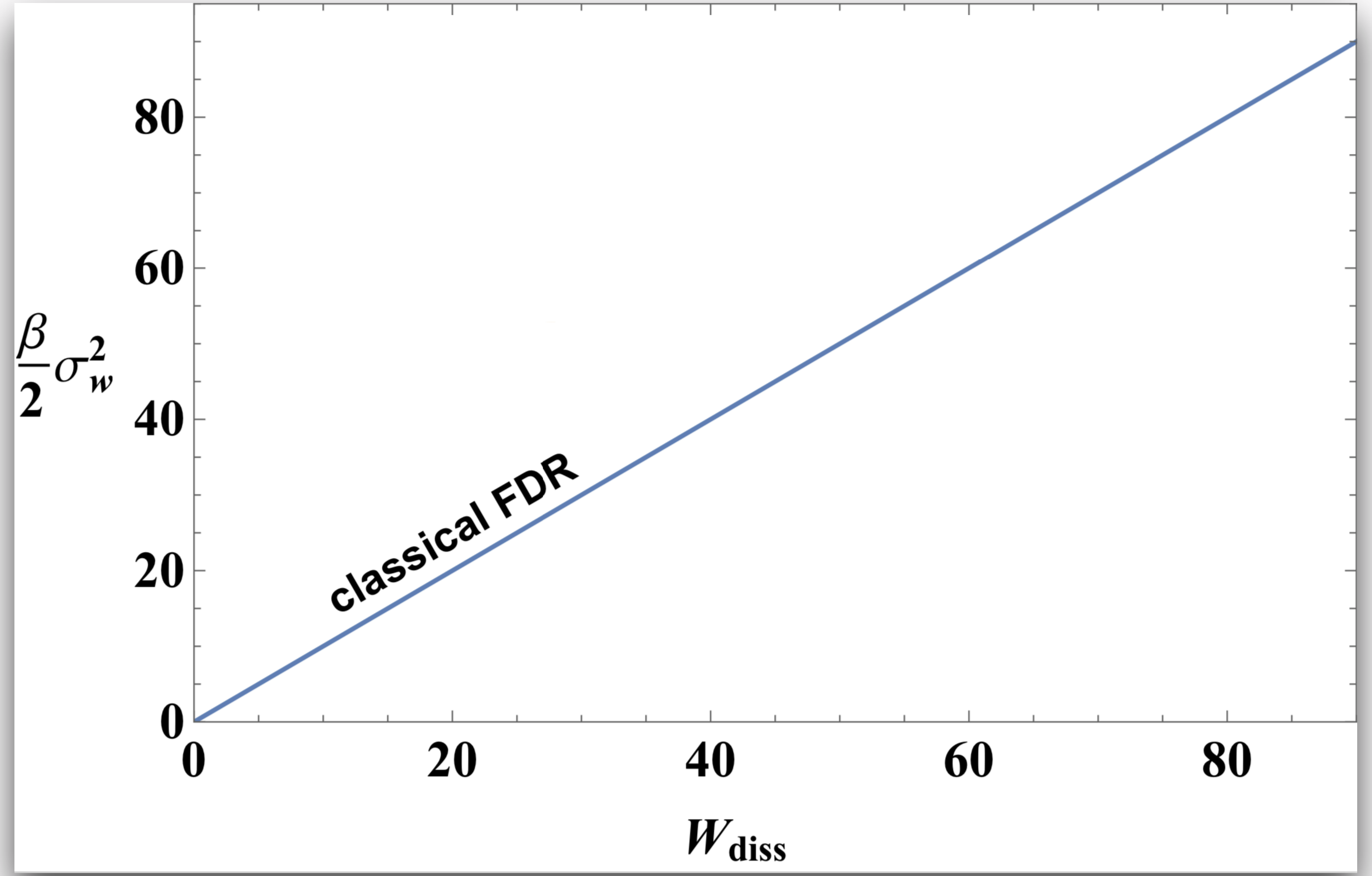

Optimise: $\mathscr{C}_\alpha = \alpha \frac{\beta}{2} \langle \sigma^2 \rangle + (1-\alpha) \langle W_{diss} \rangle$

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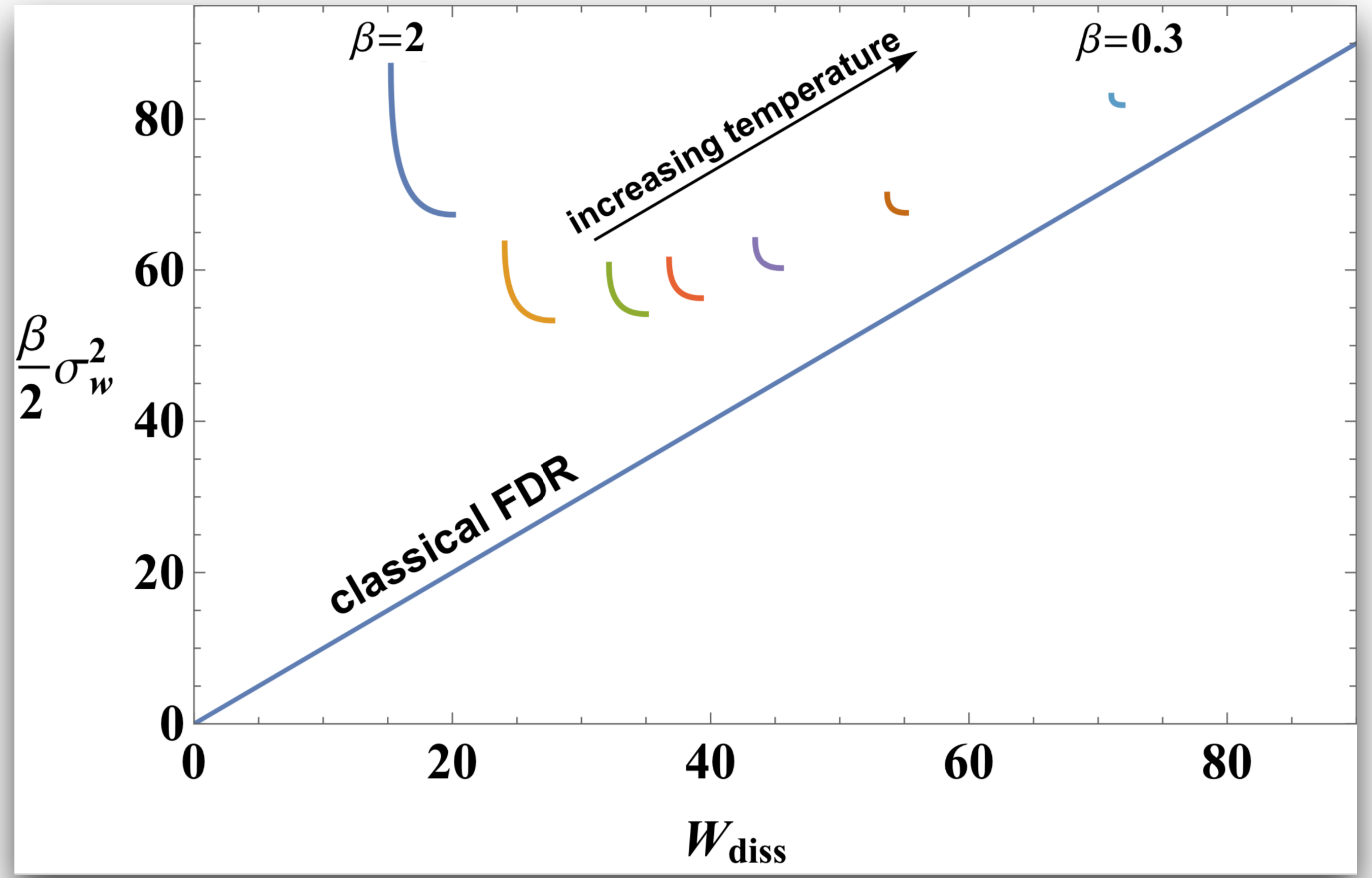

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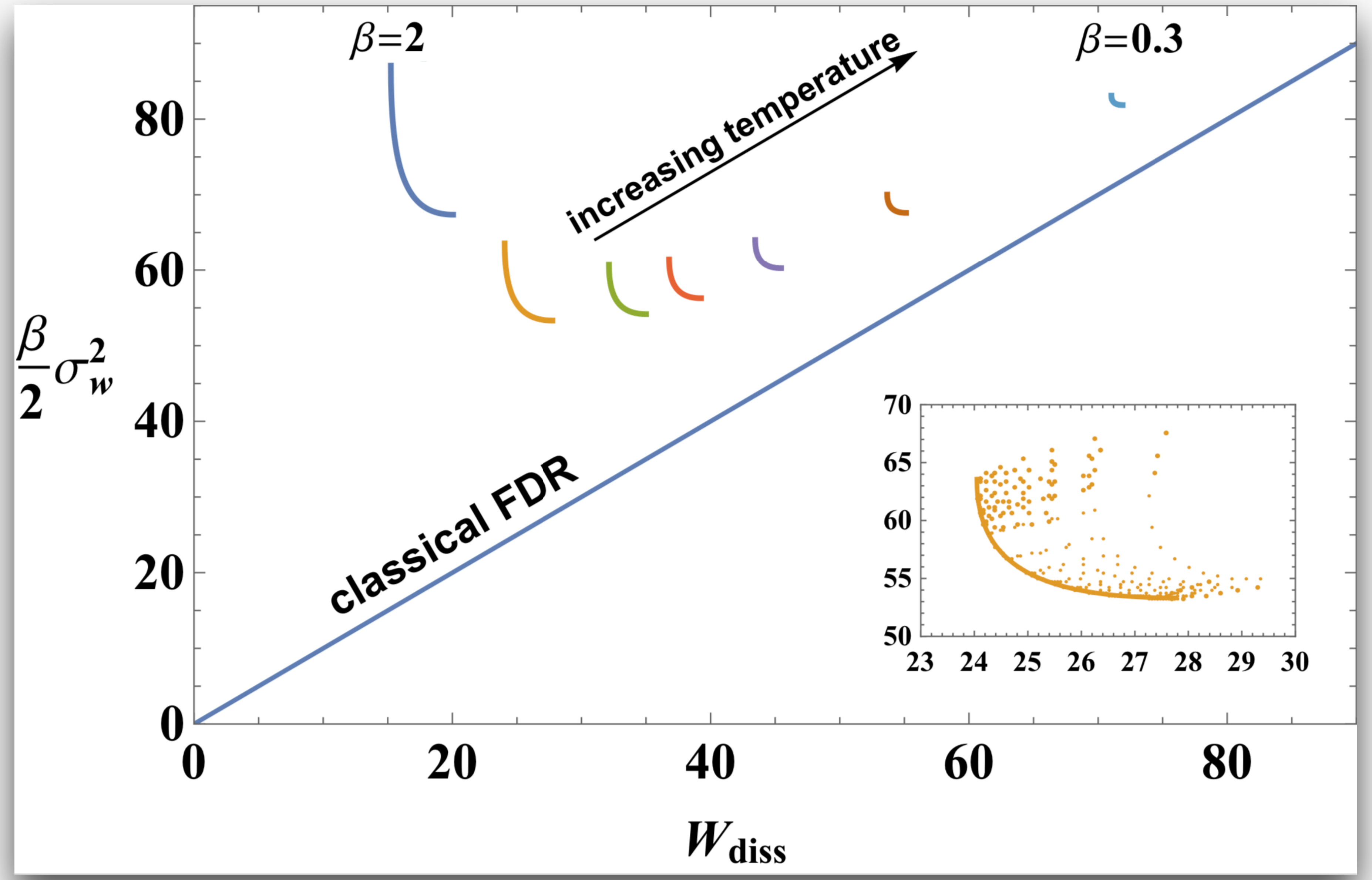

Optimise: $\mathscr{C}_\alpha = \alpha \, \frac{\beta}{2} \langle \sigma^2 \rangle + (1-\alpha) \langle W_{diss} \rangle$

Conclusions

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Quasistatic

Classical case	**Quantum case**

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Quasistatic

Classical case	Quantum case
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Conclusions

Quasistatic

Classical case	Quantum case
$\frac{\beta}{2}\langle\sigma^2\rangle = \langle W_{diss}\rangle$	$\frac{\beta}{2}\langle\sigma^2\rangle = \langle W_{diss}\rangle + \mathcal{Q}$

Conclusions

Quasistatic

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$\frac{\beta}{2}\langle\sigma^2\rangle = \langle W_{diss}\rangle$	$\frac{\beta}{2}\langle\sigma^2\rangle = \langle W_{diss}\rangle + \mathcal{Q}$
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Conclusions

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Conclusions

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$\frac{\beta}{2}\langle\sigma^2\rangle = \langle W_{diss}\rangle$	$\frac{\beta}{2}\langle\sigma^2\rangle = \langle W_{diss}\rangle + \mathcal{Q}$
Gaussian work distribution	**Non gaussian work distribution**
Unique thermodynamic length	

Conclusions

Quasistatic

Classical case	Quantum case
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Gaussian work distribution	Non gaussian work distribution
Unique thermodynamic length	A continuous family of metrics

Thanks for the attention!

arXiv:1905.07328

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